Root Locus Methods

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Problem 4.1: Design of a position control system using the root locus method

(a) State the functions of the proportional, integral and derivative parts of a PID controller.
(b) Fig. 1 shows a position control system.

$$G(s) = \frac{60}{s(s+2)}$$

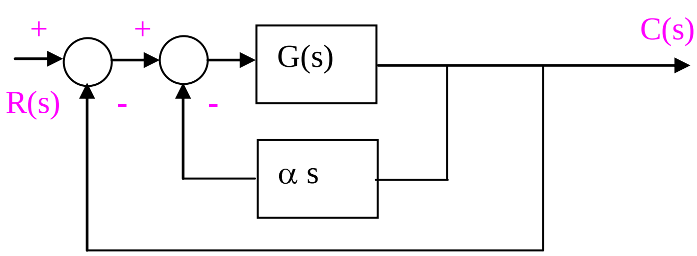


Fig.1

Determine the value of α in the velocity feedback loop so that the damping ratio ξ of the closed loop poles is 0.5.Use the root locus method.

Solution:

The characteristic polynomial of the system with velocity feedback is

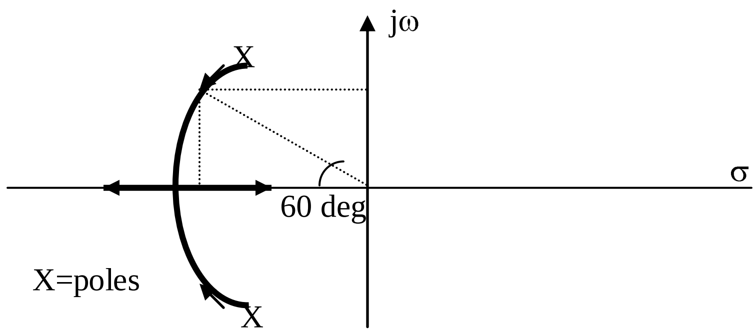
$F(s) = s^2+(60\alpha+2)s+60$

$= (s^2+2s+60)+60\alpha s$

Draw the root locus for

$$\frac{60\alpha s}{s^2+2s+60}$$

and determine α for ξ=0.5



α for desired root location is .0958 (Answer)

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Problem 4.2: Design of a phase lag compensator using the root locus method

(a) Write down the steps necessary for the design of a phase-lag compensating network for a system using the root locus method.

(b)The uncompensated loop transfer function for a system is

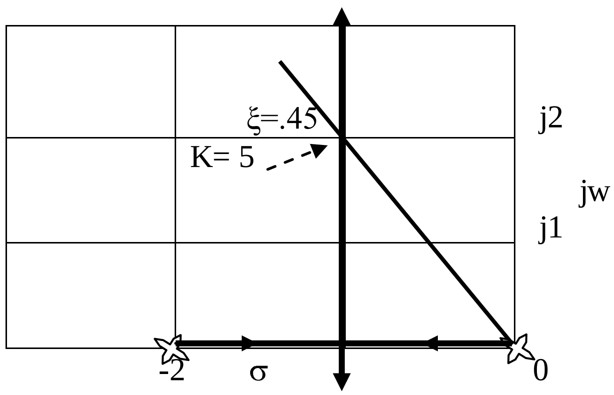
$$GH(s) = K/[s(s+2)].$$

(i) Use root locus method to design a compensator so that the damping ratio of the dominant complex roots is 0.45 while the system velocity constant, K_v is equal to 20.
(ii) Find the overall gain setting

Solution:

(a) Theory

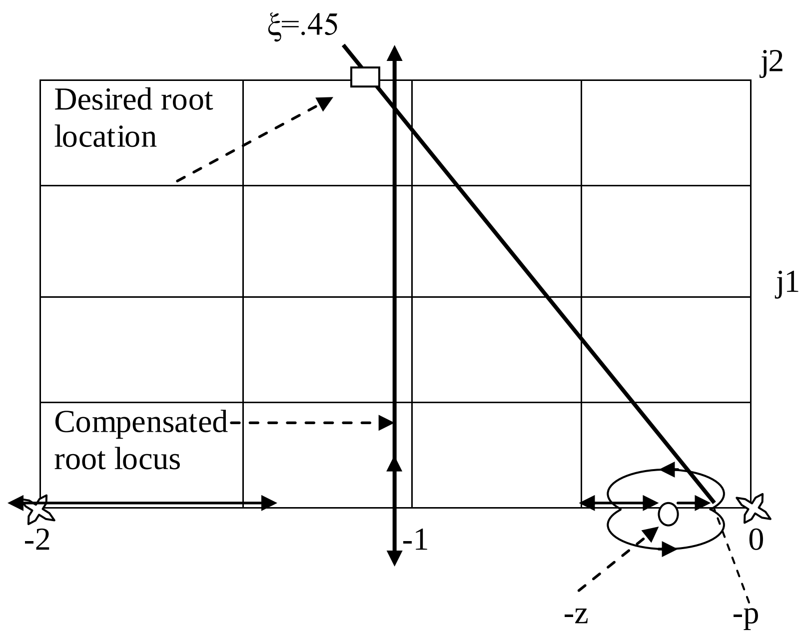
(c) Uncompensated root locus is a vertical line at s =-1 and results in a root on the ξ line at s = -1 ± j2 as shown below.



Measuring the gain at this root, we have $K = (2.24)^2 = 5$
K_v of uncompensated system = K/2 =5/2 =2.5
Ratio of zero to pole of compensator is $|z/p| = \alpha = K_{vcomp}/K_{v\,uncomp} = 20/2.5 = 8$
Examining the following Fig, we find that we might set z =-0.1 and then p = -0.1/8. The difference of the angles from p and z at the desired root is approx 1^0, and therefore s =-1+j2 is still the location of the dominant roots.

Sketch of the compensated root locus is shown below.
Compensated system transfer function is
$G_c(s)GH(s) = 6(s+0.1)/[s(s+2)(s+0.0125)]$,

where (K/α) 5 or K=40 in order to account for the attenuation of the lag network.



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Problem 4.3: The root locus procedure

i Plot the root locus for the characteristic equation of a system when

$$1+ K/[s(s+2)\,(s+2+j2)\,(s+2-j2)] = 0$$

as K varies from zero to infinity.

ii. Determine the number of separate loci.

iii. Locate the angles of the asymptotes and the centre of the asymptotes.

iv. By utilizing the Routh-Hurwitz criterion, determine the point at which the locus crosses the imaginary axis.

v. What is the limiting value of gain K for stability?

Solution: i

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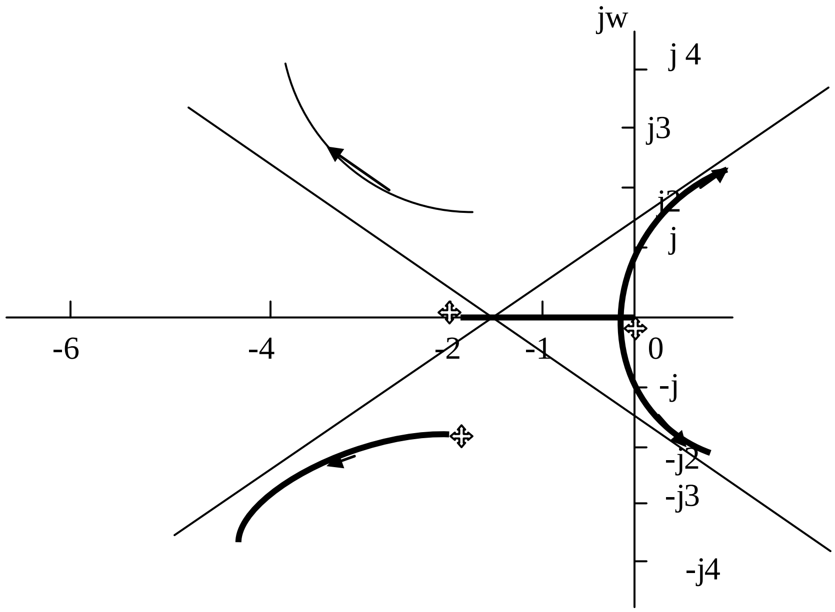

(ii) separate loci =4

(iii) Angles of asymptotes= 45,135,225,315 degs.

Centre of asymptotes= (-2-2-2)/4=-1.5

(iv)

Charac.eqn.

$s(s+2)(s^2+4s+8) + K = 0$

Apply RH criterion for stability and obtain K_{crit}.

S^4	1	12	K
S^3	6	8	
S^2	32/3	K	
S^1	8-(9K/16)		
S^0	K		

(v) K_{crit} = 14.2 and roots of the auxiliary eqn are

$10.66s^2+14.2 = 10.66(s^2+ 1.33) = 10.66(s+j1.153)(s-j1.153)$

The locus crosses the im axis at j1.153 and –j1.153

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Problem 4.4: To determine the value of the gain at the limit of stability using root locus

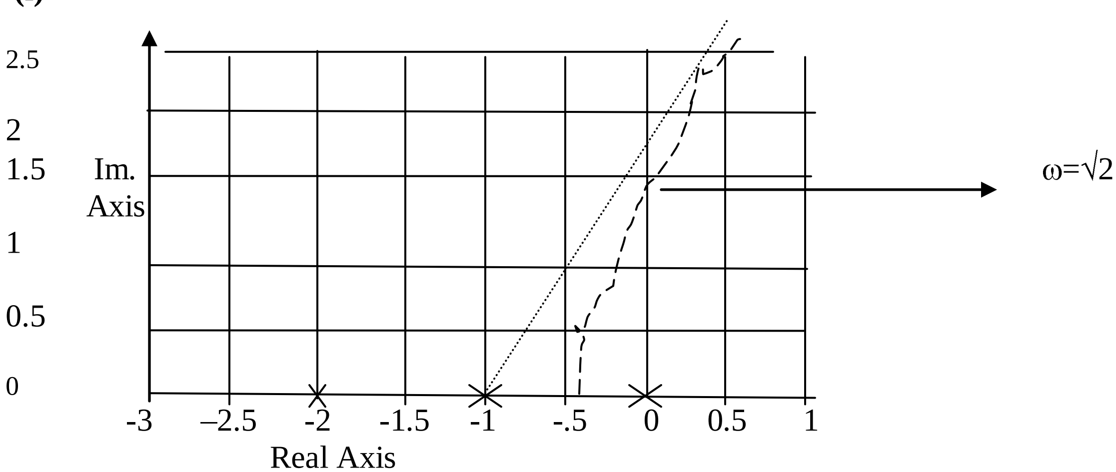
The open loop transfer function of a unity feedback system is:

$$G(s) = \frac{K}{s(s+1)(s+2)}$$

i. Draw the root locus and the asymptotes on the plot
ii. Calculate the value for the asymptote centroid.
iii. Calculate the number of asymptotes
iv. Calculate the value of ω at which the locus crosses the imaginary axis, and find the corresponding value of K at the limit of stability
v. Find the value of K required to place a real root at σ = -5.
For this value of K, find the location of the complex roots.

<u>Solution:</u>

(i)



(ii)σ_P= -3/3 = -1

(iii) 3 asymptotes @ -60^0, 60^0, -180^0

(iv) $\omega=\sqrt{2}$

K= 6 @ stability limit

(v) K= 60,

Roots are @ s= -5,1-j3.32, 1+j3.32

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Problem 4.5 : Breakaway point and the limiting value of gain for stability

Construct the root locus for the control system shown in Fig.2. Follow the following six steps:

(i) Locate the poles on the s-plane.
(ii) Draw the segment of the root locus on the real axis.
(iii) Draw the three separate loci.
(iv) Determine the angles and the center of the asymptotes.
(v) Evaluate the breakaway point for one of the loci.
(vi) Determine the limiting value of the gain K for stability using the Routh-Hurwitz criterion.

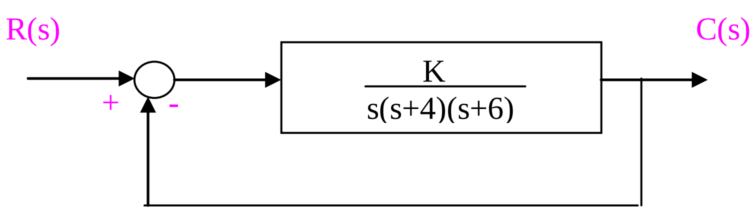
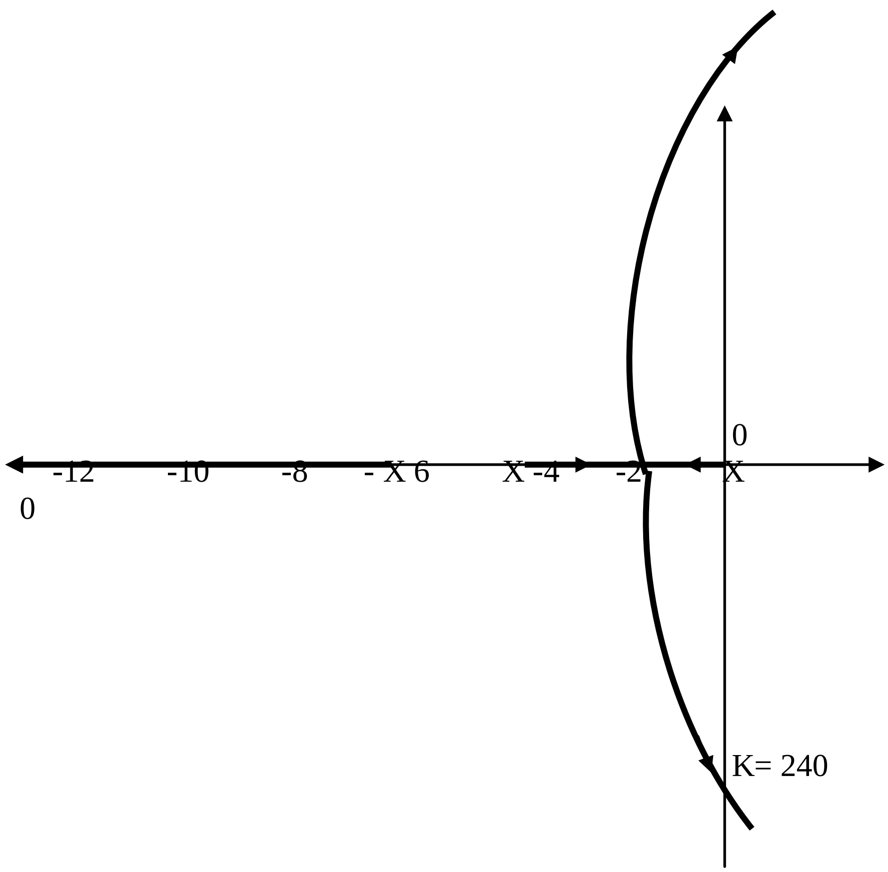


Fig.2

Solution:

-Characteristic eqn. s(s+4)(s+6)+K=0.
Poles are at s= 0, -4, -6. There are no zeros.
On the real axis, there are loci between the origin and –4 and from –6 to –infinity.
Asymptotes intersect the real axis at
σ =[0+(-4) +(-6)]/3 =-3.333
Angles of asymptotes are +/- 60 deg. and 180deg.
To determine breakaway point, note that
-K = s(s+4)(s+6) K=240
-dK/ds = $3s^2$+20s+24= 0
Therefore, the breakaway point is on the real axis at s=-1.57.



The application of Routh's criterion to the characteristic equation

$s(s+4)(s+6)+K=0$ gives the following array:

s^3	1	24	0
s^2	10	K	0
s^1	240-K/10	0	
s^0	K		

The value of K which makes the rows[1] vanish is K=240. This is the limiting value of K for stability.

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Problem 4.6: Breakaway point and the limiting value of gain for stability

Plot the complete root locus of the following characteristic equation of a system, when K varies from zero to infinity.

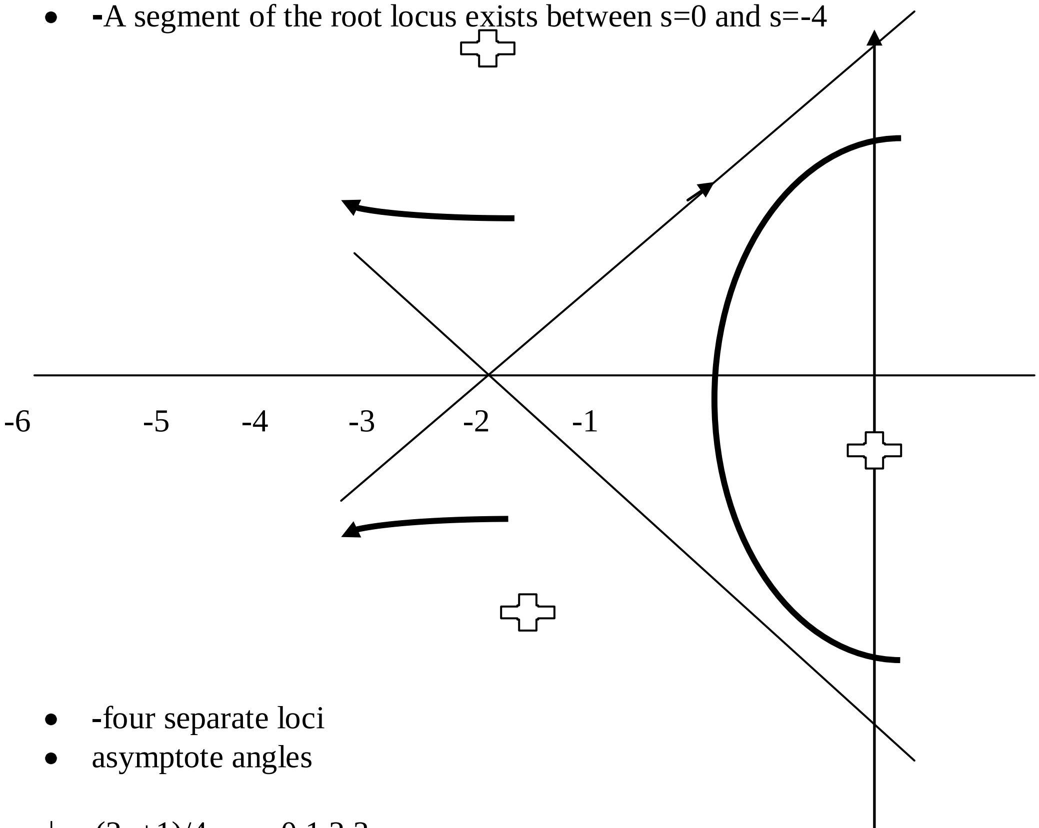
$$1 + \frac{K}{s(s+4)(s+4+j4)(s+4-j4)} = 0.$$

Follow the following seven steps:

i. Locate the poles on the s-plane
ii. Draw the segment of the root locus on the real axis
iii. Draw the four separate loci
iv. Determine the angles and the center of the asymptotes.
v. Evaluate the breakaway point for one of the four loci.
vi. Determine the limiting value of the gain K for stability using the Routh-Hurwitz criterion.
vii. Determine the angles of departure at the complex poles.

Solution:

- **-**A segment of the root locus exists between s=0 and s=-4



- **-**four separate loci
- asymptote angles

ϕ =(2q+1)/4, q =0,1,2,3

ϕ =45,135,225,315 deg.

asymptote center = (-4-4-4)/4 =-3

- Breakaway point is obtained from

K =p(s) =-s(s+4) (s+4+j4)(s+4-j4) between s= -4 and s=0

Search s for a maximum value of p(s) by trial and error . It is approximately s=-1.5

- Write the characteristic equation and Routh array. Limiting value for stability is K=570
- Angle of departure at complex poles is obtained from the angle criterion.

Angles are 90,90,135 and 225 deg.

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Problem 4.7: Root locus analysis when G(s) contains a zero

(a) Suggest a possible approach to the experimental determination of the frequency response of an unstable linear element within the loop of a closed-loop control system.

(b) Fig.3 shows a system with an unstable feed-forward transfer function.

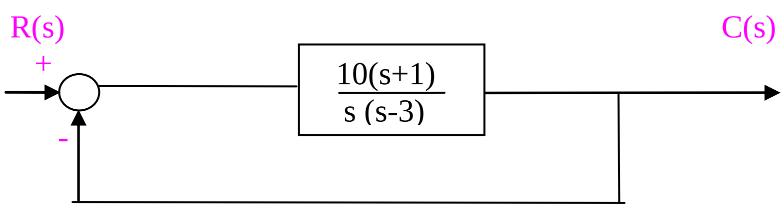


Fig.3

Sketch the root locus plot and locate the closed loop poles. Show that although the closed-loop poles lie on the negative real axis and the system is not oscillatory, the unit-step response curve will exhibit overshoot.

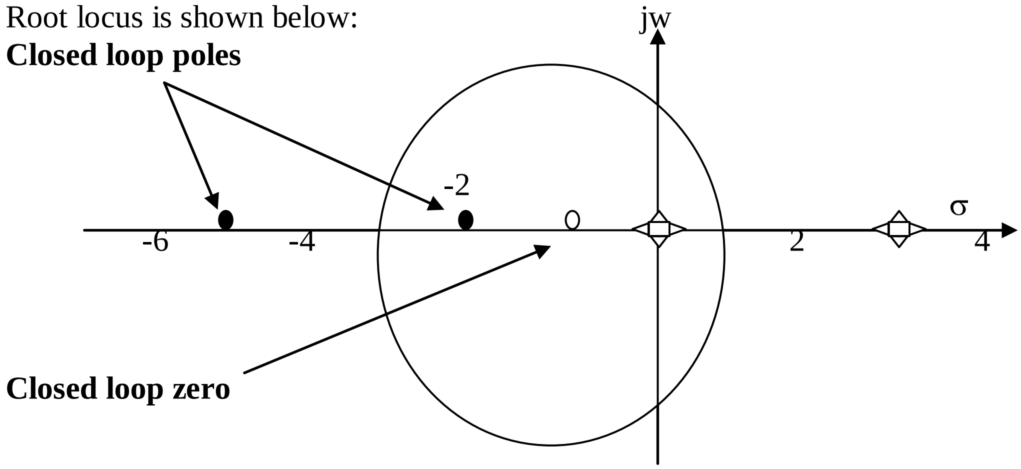
Solution:

(a)

Measure the frequency response of the unstable element by using it as part of a stable system. Consider a unity feedback system whose OLTF is G1*G2.. Suppose that G1 is unstable.. The complete system can be made stable by choosing asuiatble linear element G2. Apply a sinusoidal signal at the input.. Meausure the error signal e(t), the input to the unstable element, and x(t) the output of the unstable element. By changing the frequency and repeating the process , it is possible to obtain the frequency rsponse of the unstable element.

(b)

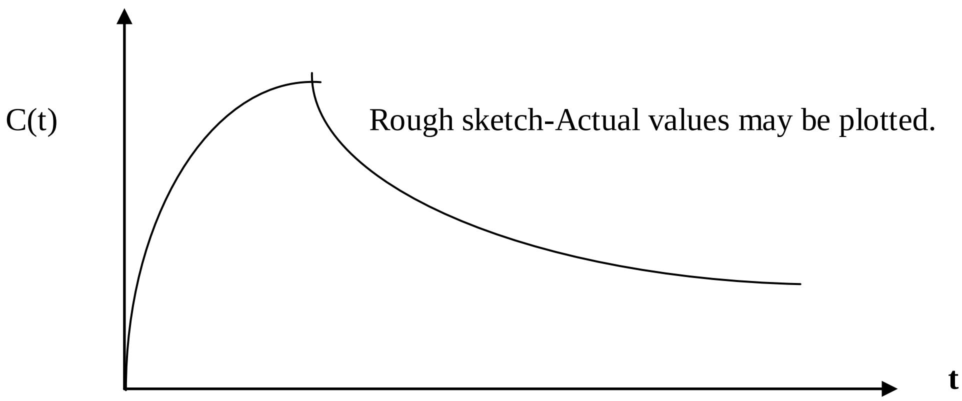
Root locus is shown below:



Closed loop transfe function is $C(s)/R(s) = \dfrac{10(s+1)}{s^2 + 7s + 10}$

Unit step response of the system is

$C(t) = 1 + 1.666\exp(-2t) - 2.666\exp(-5t)$



Although the system is not oscillatory, the response exhibits overshoot. This is due to the prsence of azero at s=-1.

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Problem 4.8: Root locus plot giving the number of asymptotes,centroid, and the frequency at which the locus crosses the imaginary axis

(a) Draw the root locus plot for a unity feedback system with

$$G(s) = \frac{K}{s(s+5)^2}.$$

Calculate and record values for the asymptote centroid, number of asymptotes, and value of the frequency, ω at which the locus crosses the imaginary axis. Also, show the asymptotes on the plot.

(b) How would you obtain qualitative information concerning the stability and performance of the system from the root locus plot?

Solution:

(a) Poles at s=0, two at s=-5. Root locus crosses imaginary axis at w=5
Centroid at –10/3=-3.33
Three asymptotes at (=/-) 60deg., -180deg
K= 250 at stability limit
(b) Descriptive

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Problem 4.9: Design the value of K in the velocity feedback loop to limit the overshoot for a step input using root locus

An ideal instrument servo shown in Fig.4 is to be damped by velocity feedback. Maximum overshoot for a step input is to be limited to less than 25%.

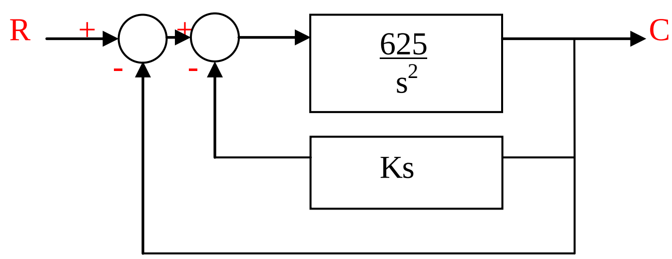


Fig.4

(a) Design the value of K using the root-locus method.
(b) Determine the step response for the designed value of K.

Solution:

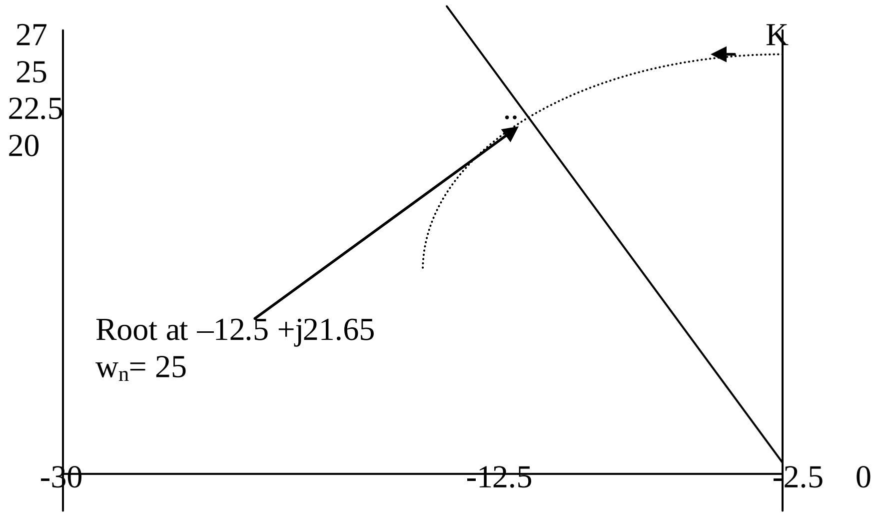
System without velocity feedback:

$$\frac{625}{(s^2+625)}, \qquad \text{roots } s=-/+\ j25$$

Open loop Bode diagram magnitude curve is a straight line with slope of –40dB/decade, crossing 0dB at w = 25.

With velocity feedback added the root locus is as shown with K as the gain.

$$OLTF = \frac{625/s^2}{\left(1+(625Ks/s^2)\right)} = \frac{625}{s(s+625K)}$$



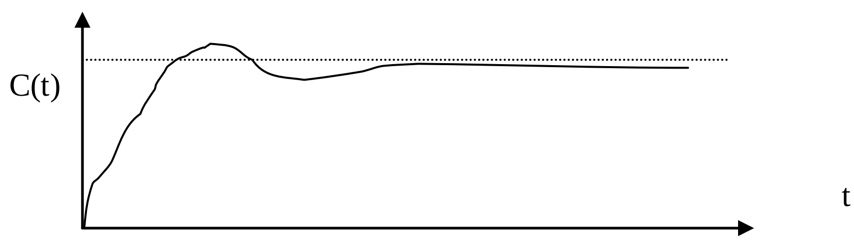
By inspection, any desired value of ξ is available. Choosing ξ =0.5 to get Mpt < 1.25.

Mpt= 1+ exp(-PI ξ/sqrt(1-ξ^2)) =1.163 for ξ =0.5

K= .04 and roots are at s= -12.5+ j 21.65

<u>Step response:</u>

$$C(t) = 1- \frac{\exp(-\xi wn\, T)}{\text{sqrt}(1-\xi^2)} \sin(\, wn\ \text{sqrt}(1-\xi^2)+ \tan^{-1}(\text{sqrt}(1-\xi^2)/\ \xi)\ \ )$$



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Problem 4.10: To determine the σ co-ordinate when the imaginary co-ordinate at some point, ω on the root locus is given

(a) Give the value of ω where the root locus of

$$G(s) = \frac{K}{(s+1)(s+1-j)(s+1+j)}$$

crosses the imaginary axis.

(b) Give the values of σ where the root locus of

$$G(s) = \frac{K(s+1)(s+5)}{s^2(s+2)}$$

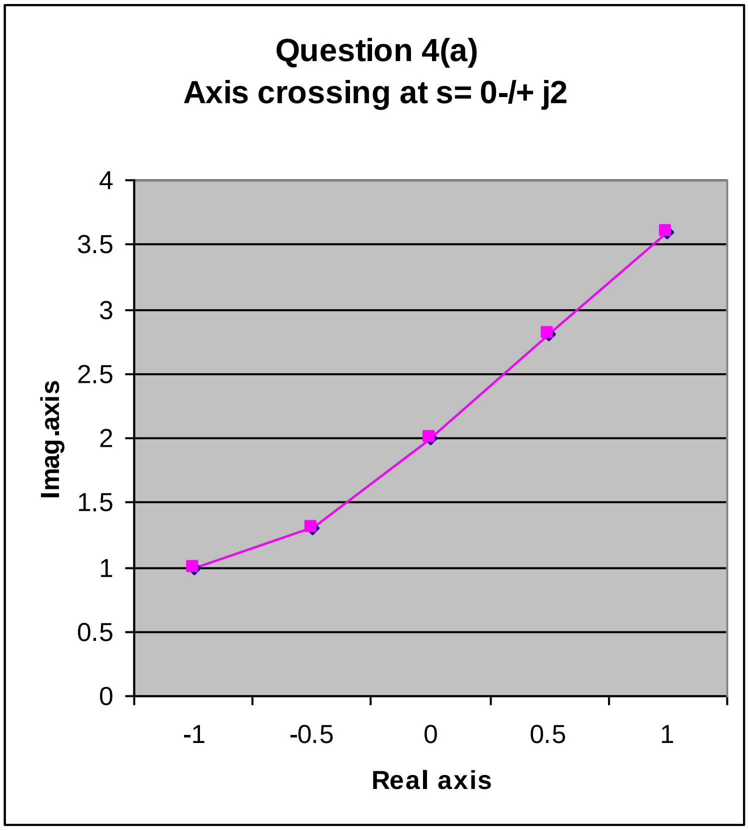
leaves and re-enters the real axis.

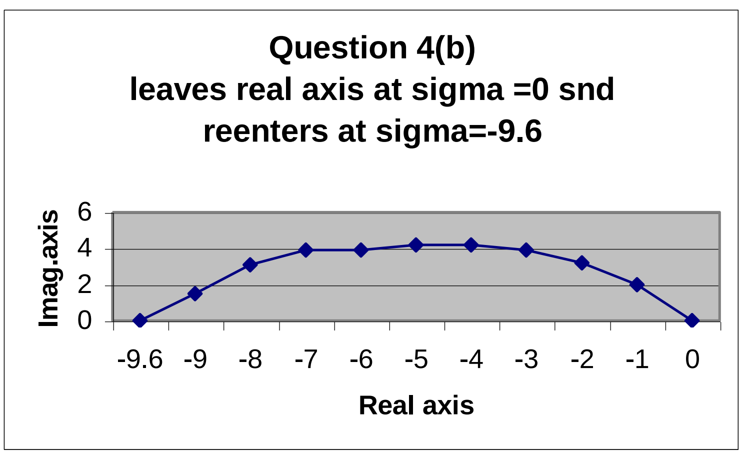
(c) Draw the root locus for

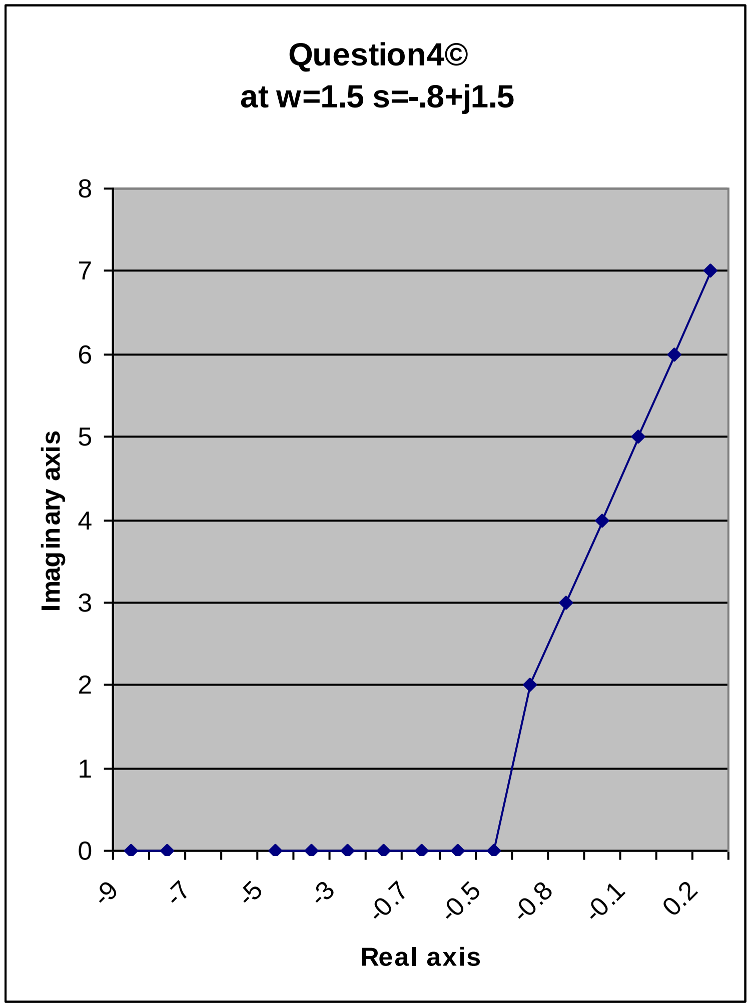
$$G(s) = \frac{K(s+2)}{s(s+1)(s+5)(s+8)}$$

What is the σ co-ordinate when the imaginary co-ordinate at some point is $\omega = 1.5$?

Solution:


Question 4(a)
Axis crossing at s= 0-/+ j2
Imag.axis
4
3.5
3
2.5
2
1.5
1
0.5
0
-1
-0.5
0
0.5
1
Real axis


Question 4(b)
leaves real axis at sigma =0 snd
reenters at sigma=-9.6
Imag.axis
6
4
2
0
-9.6
-9
-8
-7
-6
-5
-4
-3
-2
-1
0
Real axis



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Objective Type Questions

(i) The root locus in the s-plane follows a circular path when there are

(a) two open loop zeros and one open loop pole
(b) two open loop poles and one open loop zero
(c) two open loop poles and two open loop zeros

Ans: (b)

(ii) The algebraic sum of the angles of the vectors from all poles and zeros to a point on any root locus segment is

(a) 180^0
(b) 180^0 or odd multiple thereof
(c)180^0 or even multiple thereof

Ans: (b)

(iii) The algebraic sum of the angles of the vectors from all poles and zeros to the point on any root-locus segment is

(a) 180^0
(b) 180^0 or odd multiple thereof.
(c) 180^0 or even multiple thereof
(d) 90^0 or odd multiple thereof.

Ans: (d)

(iv) Whenever there are more poles than zeros in the open loop transfer function G(s), the number of root locus segments is

(a) Equal to the number of zeros
(b) Equal to the number of poles
(c) Equal to the difference between the number of poles and number of zeroes
(d) Equal to the sum of the number of poles and number of zeroes.

Ans: (b)

(v) If the number of poles is M and the number of zeroes is N, then the number of root-locus segments approaching infinity is equal to

(a) M+N
(b) M/N
(c) MN
(d) M-N

Ans:(d)

(vi) If the system poles lie on the imaginary axis of the s-plane, then the system is

(a) stable
(b) unstable
(c) conditionally stable
(d) marginally stable

Ans:(d)

(vii) The Routh array for a system is shown below:

s^3	1	100
s^2	4	500
s^1	25	0
s^0	500	0

How many roots are in the right-half plane?

(a) One
(b) Two
(c) Three
(d) Four

Ans:(b)

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