Digital Control Systems

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Problem 9.1: Impulse response of a sampled data system

Obtain the unit impulse response of the open-loop sampled data system shown in Fig.1

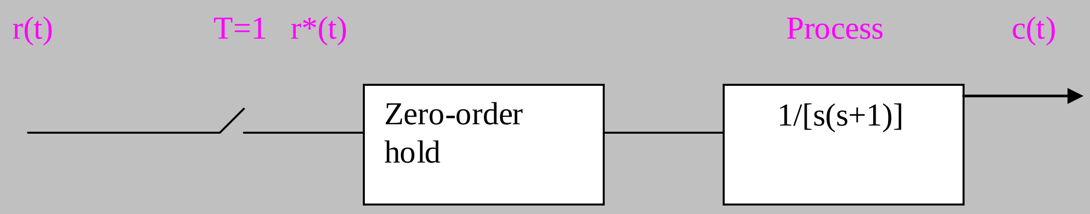

Fig.1

Solution:

From Fig.

$$G(s)=(1-\exp(-st))/[s^2(s+1)] =(1-\exp(-st))[1/s^2 - 1/s + 1/(s+1)]$$

$$G(z)= Z\{G(s)\}(1-z^2)Z\{1/s^2 - 1/s + 1/(s+1)\}$$

For T=1, we therefore have

$$G(z)= \frac{z\exp(-1)+1-2\exp(-1)}{(z-1)(z-\exp(-1))} = \frac{0.3678z+0.2644}{z^2-1.3678+.3678}$$

For unit impulse R(z) = 1 so that C(z) =G(z)

Dividing denominator into the numerator, we get

$$C(z) = .3678z^{-1} + .7675z^{-2} + .9145z^{-3} + \dots$$

The response is

C(0)=0
C(T)= .3678
C(2T)= .7675
C(3T)= .9145 etc.

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Problem 9.2: Characteristics of a digital controller (in the difference equations form) to give a specified response of the system to a unit step function.

Fig. 2 shows a digital control system.

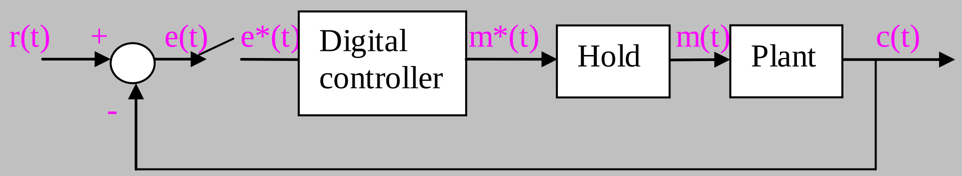

Fig. 2

(a) Show that the z transform, D(z), of the digital controller is given by

$$D(z) = [1/G(z)].[C(z)/R(z)]/[1 – \{C(z)/R(z)\}]$$

where G (z) is the z transform of zero order hold and plant.

(b)The transfer function of the plant is

$$(s+2)/[s(s+1)].$$

Determine the characteristics of a digital controller (in the difference equations form) such that the response of the system to a unit step function will be $c(t) = 5(1-e^{-2t})$. The sampling period is 1 s.
Use the following table of Laplace and z transforms.

Time function -Laplace transform-Z transform

u (t)-1/s-z/(z-1)
t-$1/s^2$-$zT/(z-1)^2$
e^{-at}-$1/(s+a)$-$z/(z-e^{-aT})$

Solution:

For the plant and zero order hold $G(s) = G1(s)G2(s) = (1- e^{-Ts})\ \ (s+2)/[s^2(s+1)]$

$G1(s) = 1-e^{-Ts}$.

$G2(s) = (s+2)/s^2(s+1) =(2/s^2)-(1/s)+(1/(s+1))$

$G(z) =[(z-1)/z]\{ [2z/(z-1)^2] –[z/(z-1)]+[z/(z-.368)]$

$=(1.368z-0.104)/(z^2-1.368z+0.368)$

For
$C(s) = 5[(1/s)-(1/(s+2))]$,

Then $C(z) = 5[((z/(z-1)) - (z/(z-0.135))] = 4.32z/[(z-1)(z-0.135)]$

$C(z)/R(z) = 4.32/(z-.135)$ and $1-(C(z)/R(z)) = (z-4.45)/(z-.135)$

$D(z) = [z^2-1.368z+.368]/[1.368z-.104].[4.32/(z-4.45)]$

$= [4.32z^2-5.91z+1.59]/[1.368z^2-6.202z+.46]$

Dividing numerator and denominator by $1.368z^2$ and cross-multiplying gives

$M(z) -4.53z^{-1}M(z)+.34z^{-2}M(z) = 3.16E(9z)-4.32z^{-1}E(z) +1.16z^{-2}E(z)$

Inverting gives the controller characteristics

$m(k) = 4.53m(k-1)-.34m(k-2)+3.16e(k)-4.32e(k-1)+1.16e(k-2)$

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Problem 9.3: Position servo operated by a sampler and a zero order hold

(a) Derive a state variable equation for a system governed by the second –order difference equation

$$c(k+2)+3\,c(k+1) +2\,c(k)$$

where u (k) is the input and c (k) is the output.

(b) Fig.4 shows a position servo operated by an input via a sampler and a zero order hold with the position c and velocity dc/dt represented by $x_1$and x_2 respectively. If the states at the sampling instants alone are required ,determine the **A** and **B** matrices in the following equation

(c)

$$\begin{pmatrix} x_1(k+1) \\ x_2(k+1) \end{pmatrix} = [\mathbf{A}] \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} + [\mathbf{B}]\, u(k)$$

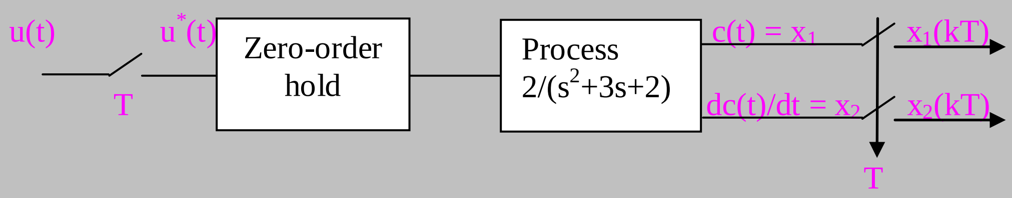

Fig.4

Solution:

(a)

Let x1(k) =c(k)

X2(k) = x1(k+10 =c(k+1)

Then x1(k+1) = x2(k)

X2(k+1) =-2x1(k)-3x2(k0+u(k)

Then in matrix form:

$$\mathbf{X}(k+1) = \begin{pmatrix} 0 & 1 \\ -2 & -3 \end{pmatrix} \mathbf{x}(k) + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u(k)$$

(b)

$$X(k+1) = \varphi(T)\, x(k) + \Psi(T)\, u(k)$$

$$\varphi(T) = L^{-1}(sI-A)^{-1} = \begin{pmatrix} s & -1 \\ 2 & s+3 \end{pmatrix}^{-1}$$

$$= \begin{pmatrix} 2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\ -2e^{-t} + e^{-2t} & -e^{-t} + 2e^{-2t} \end{pmatrix}$$

$$\text{Thus } \varphi(T) = \begin{pmatrix} 2e^{-T} - e^{-2T} & e^{-T} - e^{-2T} \\ -2e^{-T} + e^{-2T} & -e^{-T} + 2e^{-2T} \end{pmatrix}$$

$$\text{Also } \Psi(T) = \int_0^T \varphi(\ )\ b.\ d$$

$$= \int_0^T \begin{bmatrix} 2e^{-} & -e^{-2} & e^{-} & -e^{-2} \\ -2e^{-} & +e^{-2} & {}^{-}e^{-} & +2e^{-2} \end{bmatrix} \begin{bmatrix} 0 \\ \ \end{bmatrix} d$$

$$= \begin{bmatrix} 0.5 - e^{-T} + 0.5\, e^{-2T} \\ e^{-T} - e^{-2T} \end{bmatrix}$$

Setting T =1,

$$\varphi(T) = \begin{bmatrix} 0.6005 & 0.2326 \\ -.4652 & -0.0973 \end{bmatrix}$$

$$\Psi(T) = \begin{bmatrix} 0.1998 \\ 0.2326 \end{bmatrix}$$

$$x(k+1) = \begin{bmatrix} 0.6005 & 0.2326 \\ -.4652 & -0.0973 \end{bmatrix} x(k) + \begin{bmatrix} 0.1998 \\ 0.2326 \end{bmatrix} u(k)$$

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Problem 9.4: To obtain the state equation of a discrete data system described by its difference equation

(a) Write a short note on the existence and uniqueness of the solution of state equations.

(b) A discrete –data system is described by the difference equation

$$c\,(k+2) + 5c(k+1) + 3c(k) = u\,(k+1) + 2u(k)$$

where u (k) is the reference input and c (k) is the output. Show that the state equation of the system is

$$\mathbf{x}(k+1) = \begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix} \mathbf{x}(k) + \begin{bmatrix} 1 \\ -3 \end{bmatrix} u(k)$$

Assume zero initial conditions and describe the technique used to arrive at the above equation.

Solution:

(b)

Taking z-transform of the difference equation, and assuming zero initial conditions yields

$Z^2 c(z) + 5z\, c(z) + 3c(z) = zu(z) + 2u(z)$

$c(z)/u(z) = (z+2)/(z^2+5z+3)$

Characteristic eqn;

$Z^2+5z+3 = 0$

Define the state variables as $x_1(k) = c(k)$, $x_2(k) = x_2(k+1) - u(k)$,

Substitution of the state variables into the original difference eqn. gives

$x_1(k+1) = x_2(k) + u(k)$,

$x_2(k+1) = -3x_1(k) - 5x_2(k) - 3u(k)$,

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Problem 9.5: Solution of the discrete time model of a system

(a) Fig.5 shows the block diagram of a control system with feed forward and feedback paths. Apply Mason's rule to determine the transfer function, C(s)/R(s).

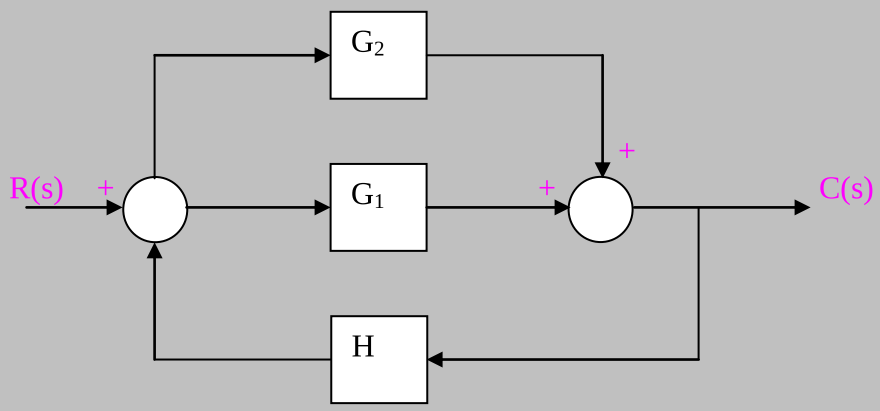

Fig.5

(b) A discrete-time model of the population of undergraduate students in an engineering college is:

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \\ x_3(k+1) \\ x_4(k+1) \end{bmatrix} = \begin{bmatrix} 0.1 & 0 & 0 & 0 \\ 0.7 & 0.1 & 0 & 0 \\ 0 & 0.82 & 0.08 & 0 \\ 0 & 0 & 0.85 & 0.02 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} u(k+1)$$

$$y(k) = \begin{bmatrix} 0 & 0 & 0 & 0.97 \end{bmatrix}$$

In the above, x_i (k) is the number of students in the ith year of the four-year degree program in academic year k, u (k) represents the input (that is, the enrollment in the first-year class in year k) and y (k) is the output (that is, the number of students graduating in year k).

In July 1992, the college had 188 students in the first year class, 126, in the second year class,

103 in the third year class, and 72 in the fourth year class.

The table below gives the history of admissions to the first year class in the subsequent years.

Year-1993-1994-1995-1996-1997-1998-1999
Admissions-275-320-350-400-450-450-430

Use this model to determine the number of students graduating in May of each year from 1994 to 2002.

Solution:

(a) There are two forward paths from R(s) to C(s), with transmittance G_1 and G_2. There is on loop with transmittance $-G_1H$, and this is touching both the forward paths. Hence,

$C(s)/R(s) = (G_1+G_2)/(1+G_1H)$

(b) The discrete –time model may be written as

x (k+1) = F.x (k) +G.u (k)
y (k+1) = C.x (k+1)

From the given data,

x (1992)=

$$\begin{pmatrix} 188 \\ 126 \\ 103 \\ 72 \end{pmatrix}$$

u (1993)=275, u (1994)=320,u (1995)= 350,u (1996)=400, u (1997)=450, u (1998)=450, u (1999)=430.

Y (1994)=C [(Fx (1992)+G.u ((1993)] = 86 (Rounding off as integer)
Y (1995)= C [(Fx (1993)+G.u ((1994)] = 94
Y (1996)= C [(Fx (1994)+G.u ((1995)] = 107
Y (1997)= C [(Fx (1995)+G.u ((1996)] = 159
Y (1998)= C [(Fx (1996)+G.u ((1997)]= 196
Y (1999)= C [(Fx (1997)+G.u ((1998)] = 220
Y (2000)= C [(Fx (1998)+G.u ((1998)] =249
Y (2001)= C [(Fx (1999)] =281
These are not affected by u (2000) and u (2001)
Y (2002)= C [(Fx (2000)] =290

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Problem 9.6: To obtain the state equation of a discrete data system described by its difference equation

(a) Derive a state variable equation for a system governed by the second-order difference equation:

c (k+2) +3c(k+1)+4c(k) = u (k)

where u (k) and c (k) are the input and output signals respectively. Write the values of the **A, B, C, D** matrices.

Draw the block diagram of the system, showing the **A, B, C, D** matrices on it.

(b) The **A, B, C, D** matrices of a single input-single-output system are: **A** = $\begin{pmatrix} 0 & 1 \\ 3 & -4 \end{pmatrix}$, **B**= $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

C= [1 0] D=0

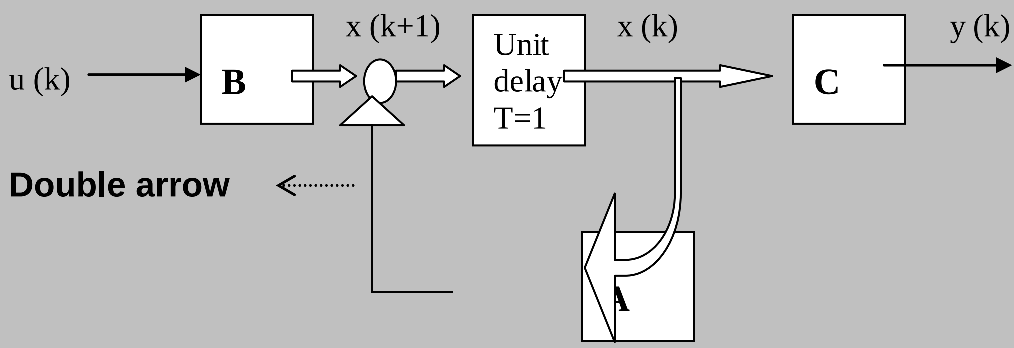
If the input is zero, and **x (0) =** $\begin{bmatrix} 0, \\ 1 \end{bmatrix}$

solve for **x (k)** and y (k), k= 1,2,3,4 etc.

<u>Solution:</u>

(a) Set x1 (k) =c (k)
and x2 (k) =x1 (k+1)=c (k+1). Then we have
X1 (k+1) = x2 (k)
X2 (k+1) =-4x1(k)-3x2(k)+u (k)
In matrix-vector form, **x** (k+1) =**Ax** (k)+**B**u (k)

And y (k) =**Cx** (k) where **A** = $\begin{pmatrix} 0 & 1 \\ 4 & -3 \end{pmatrix}$ **B**= $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ **C** = $\begin{bmatrix} 1 & 0 \end{bmatrix}$, D = 0



(b)

As u (k) = 0, we have

x (k+1) =**Ax** (k) with **x** (o)= $\begin{pmatrix} 0 \\ 1 \end{pmatrix}$

x (1) = $\begin{pmatrix} 1 \\ 4 \end{pmatrix}$

x (2) = $\begin{pmatrix} \mathbf{-4} \\ 13 \end{pmatrix}$

x (3)= $\begin{pmatrix} 13 \\ 40 \end{pmatrix}$

x (4) = $\begin{pmatrix} -40 \\ 121 \end{pmatrix}$ and y = (0, 1, -4, 13, -40 etc)

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Problem 9.7: Discrete model of the tape-drive system when a digital controller is used

(a)Fluid flowing through an orifice can be represented by the non-linear equation

$Q = K (P_1 - P_2)^{½}$

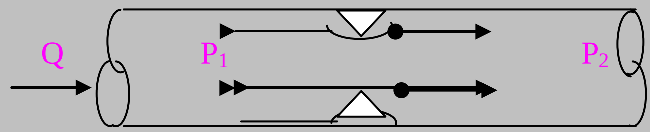

Fig.6

Fig.6 shows the variables, and K is a constant. Determine a linear approximation to the fluid flow equation.

(b) A slack loop is maintained between two sets of drive rolls, Fig.7, in a certain computer tape deck.

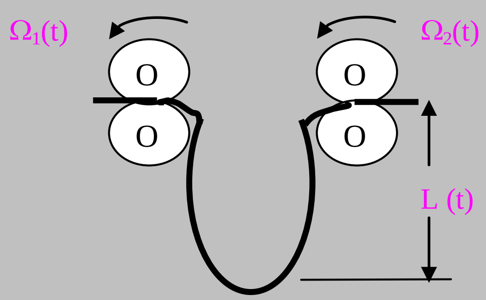

Fig.7

All rolls have diameter D. Under normal operating conditions drive speeds are $\Omega_1(t) = \Omega_2(t) = \Omega'$ and slack loop length is L (t) = L'. Starting at time t_0, there are small independent variations in the drive speeds so that $\Omega_1(t) = \Omega' + \omega_1(t)$ and $\Omega_2(t) = \Omega' + \omega_2(t)$. Derive an expression for the resulting change in the loop length l (t).

(c) Obtain a discrete model of the tape-drive system described in Part (b) when a digital controller is used,

(i) assuming that $\omega_i = (\omega_i)_{des,\ i = 1,\ 2}$.Neglect roller dynamics.
(ii) assuming that ω_i and $(\omega_i)_{des}$ are related by the differential equation $\tau d\omega_i/dt = (\omega_i)_{des} - \omega_i$, i = 1,2, where τ is the time constant of the drive motors.

Solution:

(a) $Q = K\Delta P^{1/2, \text{ where}}$ $\Delta P = P1-P2$

$\partial Q = K/(2\Delta P_0^{1/2})\ \partial P$

(b) $dL/dt = \pi D\ (\Omega_1-\Omega_2)/2 = \pi D\ (\omega_1-\omega_2)/2$

L = L' +l

Therefore, dL/dt = dl/dt

Hence, $dl/dt = \pi D\ (\omega_1-\omega_2)/2$, $l\ (t_0) = 0$

(i) The incremental model is $dl/dt = (D/2)[\omega_1(t)-\omega_2(t)]$

Due to the assumed absence of roller dynamics, $\omega_1$and ω_2 are constant over the interval $kT< t \leq (k+1)\ T$.

Therefore, $l\ (kT+T) = l\ (kT) + (DT/2)[\omega_1(kT)-\omega_2(kT)]$

(ii) Assuming first-order dynamics,

$$l\ (kT+T) = l\ (KT) + (D/2)\int_{kT}^{kT+T}[\omega_1(t)-\omega_2(t)]\ dt$$

$\omega_1(t)-\omega_2(t) = [\omega_1(kT)\text{ des}-\omega_2(kT)\text{ des}](1-e^{-\ (t-kT)/\tau})$

Therefore,

$L\ (kT+T) = l\ (kT) + = [\omega_1(kT)\text{ des}-\omega_2(kT)\text{ des}](DT/2)[1-(\tau/T)(1-e^{-T/\tau})]$

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Problem 9.8: Examples of discrete and continuous signals

State whether the following signals are discrete or continuous.

(i) elevation contours on a map
(ii) the score of basketball game
(ii) signals leaving or entering the CPU of a computer.

Solution:

(i) continuous (**ii)**discrete **(iii)**discrete

Problem 9.9: Stability of a discrete time system using bilinear transformation

(a) How would you modify the Routh-Hurwitz criterion to study the stability of a closed-loop discrete-time system?

(b) The characteristic polynomial of a closed-loop discrete-time system is given by

$z^3 + 5z^2 + 3z + 2 = 0.$

Determine the stability of the system using the bilinear transformation.

Solution:

(b)

$f(z) = z^3+5z^2+3z+2=0$

Substitute $z= (w+1)/(w-1)$ and obtain

$F(w) = 11\,w^3-w^2+w-3 = 0$

Construct Routh table. From Routh table, there is one sign change in the first column.

Hence UNSTABLE system.

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Problem 9.10: Stability from the characteristic polynomial corresponding to the pulse transfer function of a sampled data system

(a) Discuss the use of the bilinear transformation

$z = (r+1)/(r+2)$

to determine the stability of a sampled data control system.

(b) The characteristic polynomial corresponding to the pulse transfer function of a sampled data system is

$f(z) = z^3 + 5.94z^2 + 7.2\ z - 0.368$

Investigate the stability of the system.

Solution:

(b)

Using the bilinear transformation, the system characteristic equation becomes

$((r+1)/(r-1))^3+5{,}94((r+1)/(r-1))^2+7.7(r+1)/(r-1)-.368=0$

<u>or</u>

$f(\ r)=14.27r^3+2.3\ r^2-11.47r+3.13=0$

Routh's array

r^3	14.27	-11.47
r^2	2.3	3.13
r^1	-31.1	
r^0	3.13	

Two sign changes. Two zeroes of he characteristic polynomial f(z) will remain outside the unit circle. System is UNSTABLE.

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Problem 9.11: Position servo operated by a sampler and a zero order hold

(a) What is the state transition matrix? Explain its use.
(b) Fig.9 shows a sampled data position servo system with error-sampling and zero-order hold (ZOH). Fig. 10 shows its state-variable diagram. Determine θ_o and $d\theta_o/dt$ at sampling instants. Find the solution of the state vector **x(k)** for a step input θ_i (t) at sampling instants T= 1 and 2 seconds. Assume zero initial conditions. Use state transition matrix.

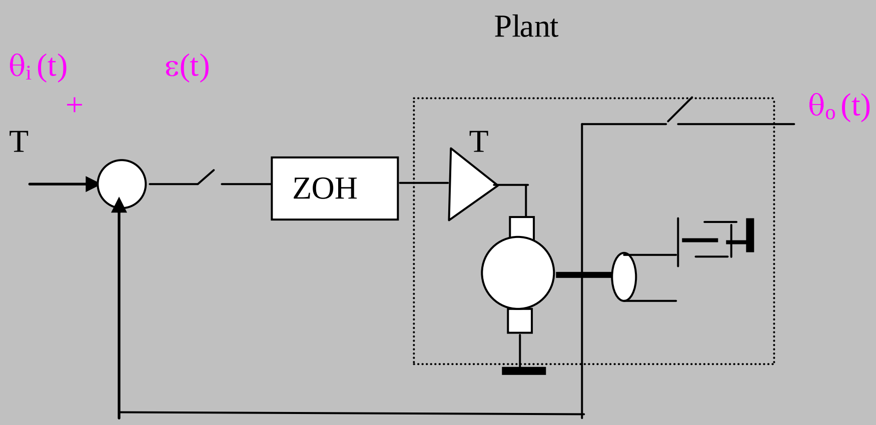

Fig.9

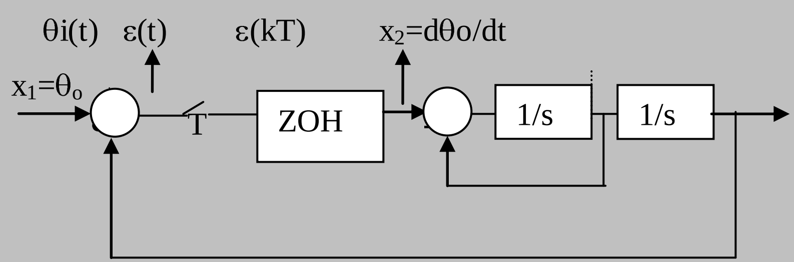

Fig.10

<u>Solution:</u>

(b)

$$A = \begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix}$$

$$B = \begin{pmatrix} 0 \\ 1 \end{pmatrix} \quad t=T \qquad \phi(T) = \text{Laplace}^{-1} \left((sI-A)^{-1} \right)$$

$$= \text{Laplace}^{-1} \begin{pmatrix} s & -1 \\ 0 & s+1 \end{pmatrix}_{t=T}$$

$$= \begin{pmatrix} 1/(s+1) & -1/(s(s+1)) \\ 0 & 1/s \end{pmatrix}_{t=T}$$

$$= \begin{pmatrix} 1 & 1-e^{-T} \\ 0 & e^{-T} \end{pmatrix} = \begin{pmatrix} 1 & .632 \\ 0 & .368 \end{pmatrix}$$

$$\psi(T) = \int_0^T \phi(\lambda) B \, d\lambda$$

$$\psi(T) = \int_0^T \begin{pmatrix} 1 & 1-e^{-\lambda} \\ 0 & e^{-\lambda} \end{pmatrix} \begin{pmatrix} 0 \\ 1 \end{pmatrix} d\lambda$$

$$= \begin{pmatrix} T + e^{-T} - 1 \\ -e^{-T} + 1 \end{pmatrix}$$

T=1

$$= \begin{pmatrix} .368 \\ .632 \end{pmatrix}$$

State variable equations become

$$X(k+1) = \begin{pmatrix} 1 & .632 \\ 0 & .368 \end{pmatrix} X(k) + \begin{pmatrix} .368 \\ .652 \end{pmatrix} [r(k)-X(k)]$$

For step function, r(k)=1, and obtain the solution

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Problem 9.12: Examples of discrete and continuous signals

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(a) State whether the following signals are discrete or continuous.

i. Temperature in a room
ii. Digital clock display
iii. The output of a loudspeaker

Solution:

i. Continuous
ii. Discrete
iii. Continuous

Problem 9.13: Drawing the simulation diagram of a system represented by the transfer function G(z)

(a) State whether the following systems are discrete or continuous:

i. Elevation contours in a map
ii. Temperature in a room
iii. Digital clock display
iv. The score of a basketball game
v. The output of a loudspeaker.

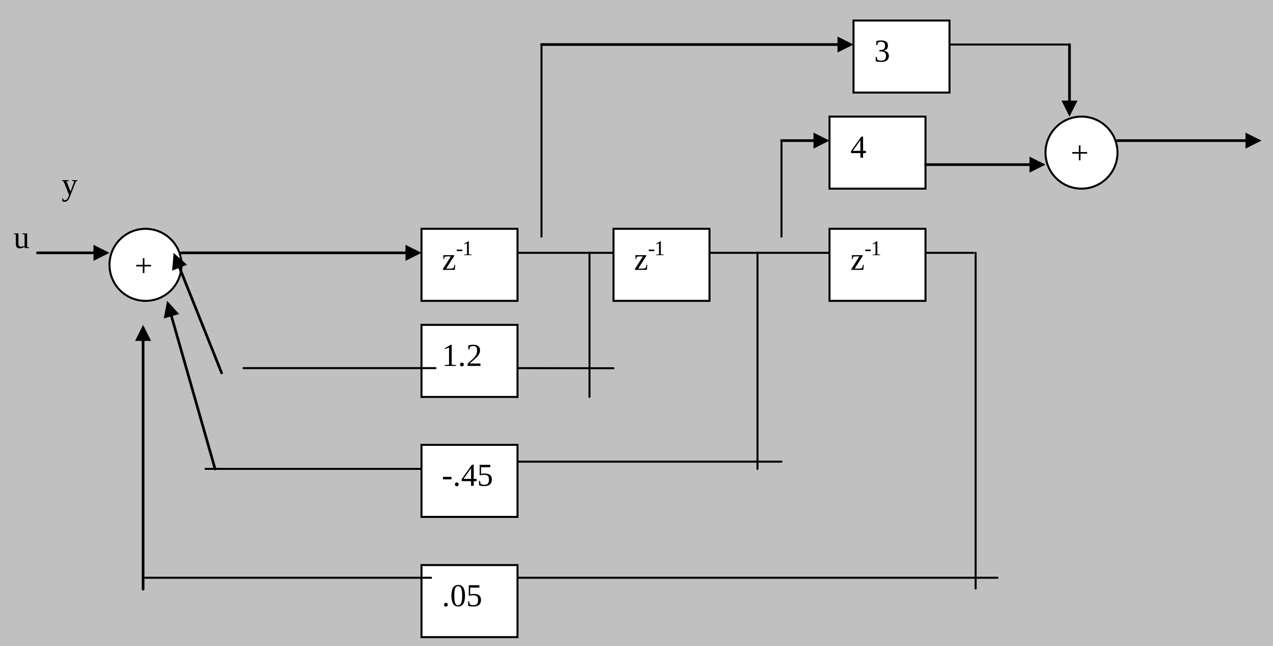
(b) A discrete –time system can be simulated on a digital computer in the same way as a continuous-time system on an analogue computer. The only difference is that integrators are replaced by delays. This implies that the blocks containing s^{-1} are replaced by blocks containing z^{-1}. Use this approach to draw the simulation diagram for a system represented by the transfer function

$$G(z) = (3z^2 + 4z)/(z^3 - 1.2z^2 + 0.45z - 0.05)$$

Solution:

$$G(z) = (3z^{-1}+4z^{-2})/(1- 1.2z^{-1} +.45z^{-2}-.05z^{-3})$$

$$=Y(z)U(z)^{-1}$$



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Problem 9.14: To obtain the equivalent discrete time equation from the state space description of a continuous time plant

(a) Describe a numerical method for the solution of continuous time state equations.

(b)The state space description for a continuous-time plant is

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 0 & -1 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Obtain the equivalent discrete-time equation. Take the sampling instant, T = 1s.

Solution:

(b) $x(t) = \exp(A(t-t0))x(t0) + \int \exp(A(t-t0))Bu(\tau)d\tau$

Setting t_0= kT and t = (k+1)T and assuming u(t) = u(kT) for kT less than/equal to t less than(k+1)T, we get

$X(k+1) = \phi(T)x(k)+\theta(T)u(k)$

where $\phi(T) = \exp(At)$

$\theta(T) = \int \exp(A\lambda)\, Bd\lambda$

For given A, $\exp(At)= \phi(t) = \begin{pmatrix} 1 & 1-\exp(-t) \\ 0 & -\exp(-t) \end{pmatrix}$

Selecting T=1s,we get $\phi(T) = \begin{pmatrix} 1 & .632 \\ 0 & .368 \end{pmatrix}$

$\theta(T) = \int \exp(A\lambda)B.d\lambda$ for limits 0 to 1 = $\begin{pmatrix} .368 \\ .632 \end{pmatrix}$

Therefore, $X(k+1) = \begin{pmatrix} 1 & .632 \\ 0 & ,368 \end{pmatrix} \begin{pmatrix} x1(k) \\ x2(k) \end{pmatrix} + \begin{pmatrix} .368 \\ .632 \end{pmatrix} u(k)$

TOP

Problem 9.15: The z-transform

(a) Show that the z-transform of f(t) = sin ωt for $t \geq 0$ is

$F(z) = (z \sin \omega t)/(z^2-2z \cos \omega t +1)$

(b) Find f(kT) if

$F(z) = 10z/(z-1)(z-2)$

(c) If $G^*(s) = \sum f(nT)\, e^{-nTs}$, show that the closed form solution for

$G(s) = 1/(s + p)$

is

$G^*(s) = \quad 1/(1-e^{T(s+p)})$

Solution:

(a) $g(t)=e^{-pt}$.

Then for t = T

$\mathbf{G^* = \sum^{\infty}_{n=0} e^{-npT} e^{-nTs}}$

$= \sum^{\infty}_{n=0} e^{-nT(p+s)}$

$= \sum^{\infty}_{n=0} X^n$ where $X= e^{-T(p+s)}$

$\sum^{\infty}_{n=0} X^n = 1/(1-X)$

$G^*(s) = 1/(1-e^{-T(p+s)})$

(b) sin wt= [exp(j wt)-exp(-j wt)]/2j

F(z) =(1/2j){z / (z-exp (jwt)) - z / (z-exp (jwt)) }

$= z \sin wt/(z^2-2z\cos wt+1)$

(c) X(z)/z = $\underline{10}$ (z-1)(z-2)

$X(z) = \frac{-10z}{(z-1)} + \frac{10z}{(z-2)}$

$x(kT) = -10 +10.2^k$

$=10(-1 +2^k)$ (k=0,1,2…)

or, x(0)=0

x(T)= 10
x(2T)= 30
x(3T)= 70
x(4T)= 150

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Problem 9.16: Pulse transfer function of a discrete time system

(a) The discrete-time system of Fig.12 has the open-loop transfer function:

$$G(s) = \frac{10}{s\,(s+1)}$$

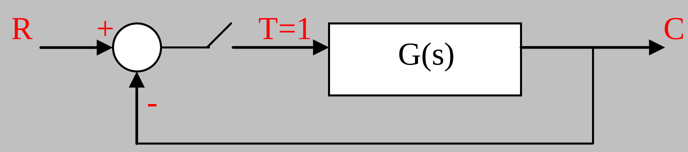


Fig.12

Show that the system is unstable.

(b) Obtain the pulse transfer function of the closed-loop system shown in Fig.13.

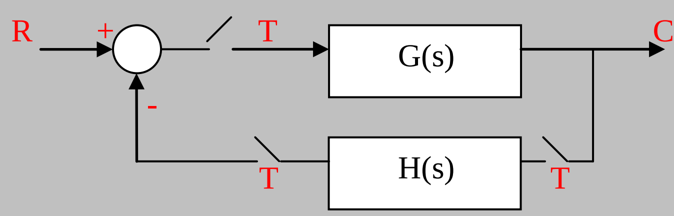


Fig. 13

Solution:

(a)

$$G(z) = \frac{10(1-\exp(-1))z}{(z-1)\,(z-\exp(-1))}$$

Characteristic equation: $1+G(z) = 0$

$(z-1)\,(z-\exp(-1)) + 10\,(1-\exp(-1))z = 0$

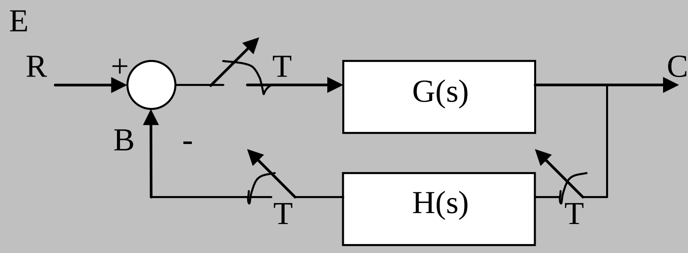
$\exp(-1)=.368$

therefore, $z^2 + 4.952z + .368 = 0$

z = 0.076, -4.876
Magnitude of one root >1

Therefore, unstable.

(b)



E(s) =R(s) – B(s)
E(z)= R(z) – B(z)
C(z) = G(z) E(z)
B(z) = H(z)C(z)
Therefore, C(z) =[R(z) –H(z)C(z)]G(z)

Or, $C(z)/R(z) = \dfrac{G(z)}{[1+H(z)G(z)]}$

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OBJECTIVE TYPE QUESTIONS:

Answer the following:

(i) In multiple- rate sampling

(a) the sampling instants are random
(b) two concurrent sampling operations occur at $t_k = pT_1$ and qT_2 where T_1 and T_2 are constants and p, q are integers
(c) the sampling instants are equally spaced, or $t_k = k T$ (k = 0, 1, 2….)
(d) the pattern of the t_k is repeated periodically

Ans: (b)

(ii) Match List E with List F given below.

List E		List F	
A	Analogue controller	I	Are high performance controllers and are combinations of analogue & digital controllers.
B	Digital controller	II	Represent the variables in the equations by continuous physical quantities and can be designed that will serve as non-decision making controllers
C	Hybrid controller	III	Operate only on numbers and are currently being used for the solution of optimal operation of industrial plants.

The correct matching is

(a)AII BIII CI
(b) AI BII CIII
(c) AIII BII CI
(d)AI BIII CII

Ans: (a)

(iii) The z-transform of $f(t) = e^t$ is

(a) z/(z-1)
(b) $z/(z-e^{T})$
(c) $z/(z- e^{-jT})$
(d) $T z/(z-1)^2$

Ans: (b)

(iv) The initial and final values of the time function corresponding to the z-transform $(4z^3-5z^2+8z)/((z-1)(z-0.5)^2)$ are

(a) Zero and indeterminate
(b) 2 and 14
(c) 4 and 28

(d) 8 and 56

respectively.

Ans: (c)

(v) Match List E with List F in the following Table of z transforms.

List E, List F
A, $x(t)=\delta(t)$, I, $x(z)= z/(z-1)$
B, $x(t)=u(t)$, II, $x(z)=Tz/(z-1)^2$
C, $x(t)=t$, III, $x(z)=z/(z-e^{-T})$
D, $x(t)=e^{-t}$, IV, $x(z)=1$

The correct matching is
(a)AIII BII CIV DI
(b)AIV BI CII DIII
©AII BIII CI DIV
(d)AI BIVCIII DII

Ans: (b)

(vi) Match List E with List F in the following Table of transducer types.

List E, List F

A, Sampled data transducer, I, A transducer in which the input signal is a quantized signal and the output signal is a smoothed continuous function of time.
B, Digital transducer, II, A transducer in which the input signal is a continuous function of time and the output signal is a quantized signal which can assume only certain discrete levels
C, Analogue to Digital transducer, III, A transducer in which the input and output signals occur only at discrete instants of time, but the magnitudes of the signals are un-quantized
D, Digital to Analogue transducer, IV, A transducer in which the input and output signals occur only at discrete instants of time, and the signal magnitudes are quantized

The correct matching is

(a)AIII BII CIV DI
(b)AIV BI CII DIII
©AIII BIV CII DI
(d)AI BIVCIII DII

Ans: (c) TOP