State Variable Analysis

State equations of a mechanical system	Optimal control system based on quadratic performance index	Transfer function of the system from the state equations	State equations of the system from the transfer function	Damping ratio of the system by minimizing the quadratic performance index
Discrete time state equation from the continuous time state equation	Equivalent electrical network and state equations for a mechanical system	Controllability, observability, and stability from the state equations	State equations of the system from the state diagram	State diagram of the system from the state equations
Analogue simulation of the system from the transfer function	Damping ratio of the system by minimizing the quadratic performance index	Transfer function of the system from the state equations	Solving the state equation	Design of a state variable feedback controller
State variable model of a hoist	Design of the observer for given observer poles	State equations of a d.c shunt motor	Transfer function of the system from the state diagram	Design of a state variable feedback controller
Transfer function of the system from the state diagram	The state transition matrix			**Objective type questions:**

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Problem: 7.1 State equations of a mechanical system

(a) Fig.1 shows a mechanical system, where

F (t) =Force applied to the system
M= Mass
B= Coefficient of friction
K= Spring constant
y= Displacement of the mass.

Write the state equations of the system, choosing displacement and velocity of the mass as state variables, x_1 and x_2, respectively.

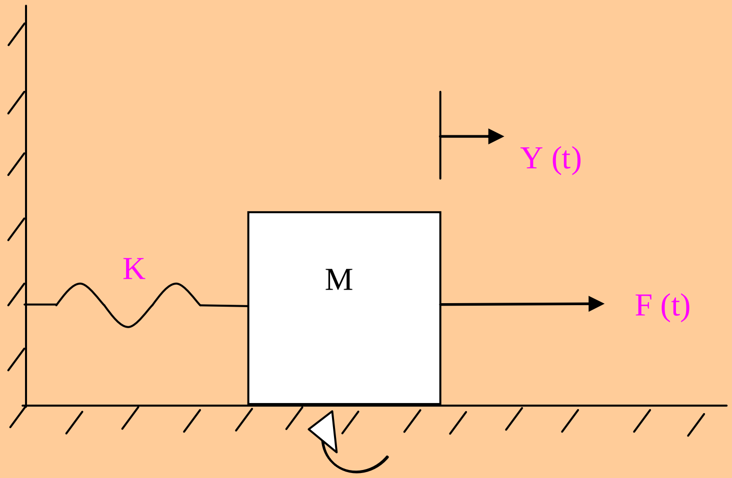

Fig.1

(b) Write the difference equation obeyed by a sum of money in a bank account at a yearly interest rate of R percent, compounded every T years. Denote the amount immediately following the kth interest period as A (kT). Find the differential equation which results as $T \rightarrow 0$.

Solution

(a) Force absorbed by mass, $f_M = Md^2y/dt^2$

Force absorbed by friction, $f_B = Bdy/dt$

Force absorbed by spring, fs= Ky

$f_M + f_B + f_s = F$

Let x1 = y, x2 = dy/dt

The state equations are:

dx1/dt = x2

dx2/dt = -Kx1/M –Bx2/M +F (t)/M

(b) The principal obeys the difference equation

A (kT+T) =A (kT) [1+RT/100].

Re-writing, [A (kT+T)-A (kT)]/T = RA (kT)/100.
Taking the limit as T tends to $\longrightarrow$ 0 and letting kT tend to $\rightarrow$ t gives the differential equation

dA/dt =RA (t)/100.

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Problem: 7.2 Optimal control system based on quadratic performance index

(i) A control system is described by

$d\mathbf{x}/dt = \mathbf{A}\mathbf{x} + Bu$

where $\mathbf{x} = \begin{pmatrix} x_1, \\ x_2 \end{pmatrix}$ $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $\mathbf{B} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

Assume the linear control law:

$U = -\mathbf{K}^t x = -k_1x_1 - k_2x_2.$

Note that $\mathbf{K}^t = \begin{pmatrix} -k_1 & -k_2 \end{pmatrix}$.

Determine the constants k_1 and k_2 so that the following performance index is minimized:

$$J = \int_0^{\infty} \mathbf{x}^t\mathbf{x}\, dt$$

The initial condition is $\mathbf{x}(0) = \begin{pmatrix} \sqrt{2} \\ 0 \end{pmatrix}$

Choose the undamped natural frequency to be 2 rad/s.

(ii) Will the optimal control law of Part (i) for the system change if the initial conditions are changed?

Solution

$$dx/dt = Ax - BK^t x = \begin{pmatrix} 0 & 1 \\ -k1 & -k2 \end{pmatrix} \begin{pmatrix} x1 \\ x2 \end{pmatrix} \qquad (1)$$

$$A - BK^t = \begin{pmatrix} 0 & 1 \\ -k1 & -k2 \end{pmatrix}$$

Eliminating x2 from eqn (1) yields

$d^2x1/dt^2 + k2dx1/dt + k1x1 = 0$. Since undamped natural frequency = 2, we obtain, k1=4.

$$\text{Therefore } A - BK^t = \begin{pmatrix} 0 & 1 \\ -4 & -k2 \end{pmatrix}$$

$A-BK^t$ is a stable matrix if k2 >0.

$J = x^t(0)\, Px(0)$ is minimized where

$P = (A-BK^t)^t P + P(A-BK) = -1$

Or,

$$\begin{pmatrix} 0 & -4 \\ 1 & -k2 \end{pmatrix} \begin{pmatrix} p11 & p12 \\ p12 & p22 \end{pmatrix} + \begin{pmatrix} p11 & p12 \\ p12 & p22 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -4 & -k2 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

Therefore,

$$\begin{pmatrix} p11 & p12 \\ p12 & p22 \end{pmatrix} = \begin{pmatrix} (5/2k2) + (k2)/8 & 1/8 \\ 1/8 & 5/(8k2) \end{pmatrix}$$

$$\mathbf{J = x^t(0)\, P\, x(0)} = \begin{pmatrix} \text{sqrt}(2) & 0 \end{pmatrix} \begin{pmatrix} p11 & p12 \\ p12 & p22 \end{pmatrix} \begin{pmatrix} \text{sqrt}(2) \\ 0 \end{pmatrix}$$

$$= (5/k2) + (k2/4)$$

To minimize J, differentiate J w.r.t k2 and set it = 0.

$dJ/dk2 = (-5/k2^2) + (1/4)$

Therefore , K2 = sqrt (20)

J_{min} = sqrt (5).

The designed system has the control law

U = -4x1- sqrt (20). x2

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Problem : 7.3 Transfer function of the system from the state equations

(a)

(i) Write the following differential equation

$$d^3y/dt^3 + d^2y/dt^2 + 3\,dy/dt + 2y = f(t)$$

in the state variable form by choosing the state variables:

$$x_1 = y,\; x_2 = dy/dt, \text{ and } x_3 = d^2y/dt^2$$

and the input variable as f (t).

(ii) Is the obtained state variable form in Part (i) a unique description of the differential equation?

(b) A control system is represented by the following state variable equations:

$$d\mathbf{x}/dt = \mathbf{Ax} + \mathbf{B}u$$
$$y = \mathbf{Cx}$$

where $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$, $\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -4 & -5 \end{pmatrix}$

$\mathbf{B} = \begin{pmatrix} 1 \\ -1 \\ 5 \end{pmatrix}$, $\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$

(i) Determine the transfer function of the system.

(ii) Determine the eigenvalues of the **A** matrix.

Solution

(a)

(i) $d^3y/dt^3 + d^2y/dt^2 + 3\,dy/dt + 2y = f(t)$

$$\begin{bmatrix} dx_1/dt \\ dx_2/dt \\ dx_3/dt \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 3 & 2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} .f(t)$$

(ii) No

(b) (i)

$$G(s) = C\,(sI-A)^{-1}\,B$$

$$(sI - A) = \begin{pmatrix} s & -1 & 0 \\ 0 & s & -1 \\ 0 & 4 & s+5 \end{pmatrix}$$

$$G(s) = (s^2 + 4s + 4) / (s^3 + 5s^2 + 4s)$$

(ii) eigenvalues are 0,-1 and -4.

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Problem : 7.4 State equations of the system from the transfer function

(a) A system is described by the following equations:

$$d\mathbf{x}/dt = \mathbf{Ax} + \mathbf{Bu}$$
$$\mathbf{y} = \mathbf{Cx} + \mathbf{Du}$$

where

$$\mathbf{A} = \begin{pmatrix} -1 & 1 \\ 0 & -2 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 1 & 2 \\ 1 & 0 \\ 1 & 1 \end{pmatrix}$$

In addition, **D** is a null matrix of proper order. Find the transfer function matrix, [Y(s)/U(s)].

(b) The overall transfer function of a single input single output control system is given by

$$Y(s)/U(s) = (s^2+4s+4)/(s^3+5s^2+4s)$$

i Find the **A**,**B** and **C** matrices in the state variable description of the system

ii Is the description of Part (i) above unique?

Solution

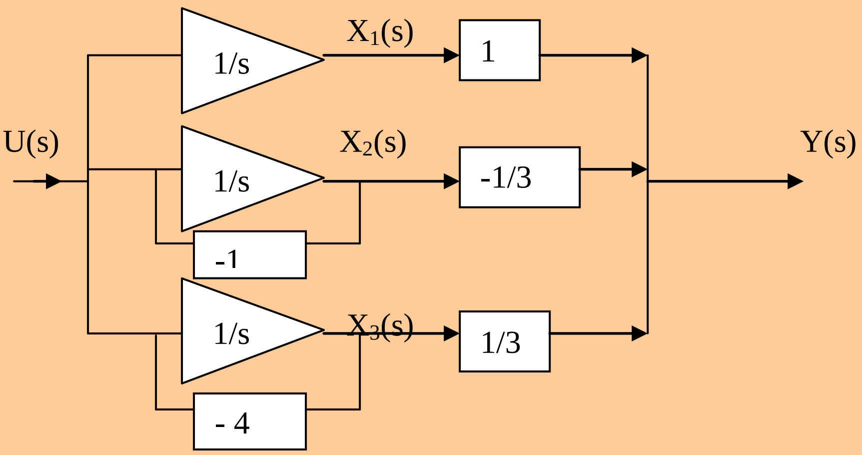
(a) $(sI-A)^{-1} = \begin{pmatrix} s+1 & -1 \\ 0 & s+2 \end{pmatrix}^{-1} = \begin{pmatrix} 1/(s+1) & 1/(s+1)(s+2) \\ 0 & 1/(s+2) \end{pmatrix}$

Transfer function, $C(sI-A)^{-1}B+D = \begin{pmatrix} 1/(s+1) & (2s+3)/(s+1)(s+2) & (3s+5)/(s+1)(s+2) \\ 1/(s+1) & 1/(s+1)(s+2) & (s+3)/(s+1)(s+2) \\ 1/(s+1) & 1/(s+1) & 2/(s+1) \end{pmatrix}$

(b)

$Y(s)/U(s) = [(s+2)^2]/[s(s+1)(s+4)] = (1/s) - [(1/3)/(s+1)] + [(1/3)/(s+4)]$

This may be represented in the block diagram form as follows:



Choosing the state variables as shown above, we write

$dx1/dt = u$
$dx_2/dt = x_2+u$
$dx_3/dt = -4x_3+u$

and $y = x_1 - x_2/3 + x_3/3$

Therefore, $A = \begin{pmatrix} 0 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -4 \end{pmatrix}$ $B = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ $C = \begin{pmatrix} 1 & -1/3 & 1/3 \end{pmatrix}$

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Problem : 7.5 Damping ratio of the system by minimizing the quadratic performance index

The control system of Fig.2 is assumed to be at rest initially.

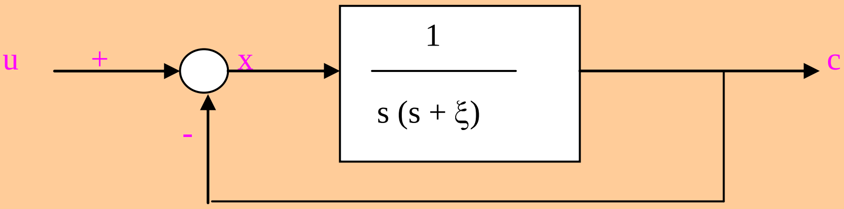

Fig.2

Determine the damping ratio $\xi > 0$ so that, for unit-step input u (t), the following performance index is minimized:

$$J = \int_0^{\infty} \mathbf{x}^T (t)\, \mathbf{Q}\, \mathbf{x} (t)\, dt$$

where

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} x \\ dx/dt \end{pmatrix}, \quad \mathbf{Q} = \begin{pmatrix} 1 & 0 \\ 0 & \mu \end{pmatrix} \quad (\mu > 0)$$

What is the optimal value of the damping ratio if $\mu = 1$?

Solution

$d^2c/dt^2 + \xi\, dc/dt + c = u$

Noting that

$x = u - c$, $u = 1$, the initial conditions are equal to zero, we have

$d^2x/dt^2 + \xi\, dx/dt + x = 0 \ (t > 0)$

The state-space representation of this eqn. becomes

$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ -1 & -\xi \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} , \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$

$$\begin{pmatrix} x_1(0) \\ x_2(0) \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

or, $d\mathbf{x}/dt = \mathbf{A}\mathbf{x}$

where $\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & -\xi \end{pmatrix}$, $J = \mathbf{x}^t(0)\,\mathbf{P}\mathbf{x}(0)$, where $\mathbf{P}$ is determined from

$$\mathbf{A}^t\mathbf{P} + \mathbf{P}\mathbf{A} = -\mathbf{Q}$$

Or, $\begin{pmatrix} 0 & -1 \\ 1 & -\xi \end{pmatrix} \begin{pmatrix} p11 & p12 \\ p12 & p22 \end{pmatrix} + \begin{pmatrix} p11 & p12 \\ p12 & p22 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ -1 & -\xi \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -\mu \end{pmatrix}$

or,

$-2p12 = -1$

$-p22 + p11 - \xi p12 = 0$

$2p12 - 2\xi p22 = -\mu$

Therefore, $P = \begin{pmatrix} (\xi/2) + \frac{1+\mu}{2\xi} & 0.5 \\ 0.5 & \frac{1+\mu}{2\xi} \end{pmatrix}$

$$J = \mathbf{x}^t(0)\,\mathbf{P}\mathbf{x}(0) = p11 = (\xi/2) + \frac{1+\mu}{2\xi}$$

To minimize J, $dJ/d\xi = 0.5 - \frac{(1+\mu)}{2\xi^2} = 0$ giving $\xi = \text{sqrt}(1+\mu)$

If $\mu = 1$, the optimal value of ξ = sqrt (2).

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Problem : 7.6 Discrete time state equation from the continuous time state equation

(a) Obtain the solution of the state equation

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \end{bmatrix} u$$

where u (t) is a unit step. The initial conditions are $\mathbf{x}^T(\mathbf{0}) = [0 \quad 1]$.

(Note that ' T ' is the transpose of the vector **x**)

(c) The state equation for a continuous-time plant is
(d)

$$\begin{bmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u$$

Obtain the equivalent discrete–time equation, suitable for a time step, T , of 1 sec.

Solution

(a)

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = 2 \begin{bmatrix} e^t + te^t - 1 \\ e^t - 0.5 \end{bmatrix}$$

Answer

(b) The solution of the equation is

$$\mathbf{X}(t) = \exp(\mathbf{A}(t-t0))\,\mathbf{x}(t0) + \int_{To}^{t} \exp(\mathbf{A}(t-\tau))\,\mathbf{B}u(\tau)\,d\tau$$

where **A** and **B** are known.

Setting t0 = kT and t = (k+1) T and assuming u (t) = U(kT) for $kT \leq t<(k+1)T$, we get

$$x(k+1) = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = 2 \begin{pmatrix} e^t \quad +te^t \; -1 \\ e^t \quad -0.5 \end{pmatrix}$$

Answer

(T) x (k) + θ(t) u (k)

where $\phi(T) = \exp(\mathbf{A}T)$, $\theta(T) = \int_0^{\tau} \exp(\mathbf{A}\lambda)\,\mathbf{B}d\lambda$

For given **A,**

$$\exp(At) = \begin{pmatrix} 1 & 1-e^{-t} \\ 0 & e^{-t} \end{pmatrix}$$

Setting T= 1 sec. we get

$\phi(T) =$

$$\exp(At) = \begin{pmatrix} 1 & 0.632 \\ 0 & 0.368 \end{pmatrix}$$

Setting T= 1 sec. we get

$\theta(T) = \int \exp(\mathbf{A}\lambda)\,\mathbf{B}d\lambda$

$$\begin{pmatrix} \\ \\ \end{pmatrix}$$

$$\int_0^t (1-\exp(-\lambda))\, d\lambda$$

$$= \int_0^t \exp(-\lambda)\, d\lambda$$

Setting T= 1 sec. we get

$$= \begin{pmatrix} 0.368 \\ 0.632 \end{pmatrix}$$

Therefore,

$$x(k+1) = \begin{pmatrix} 1 & 0.632 \\ 0 & 0.368 \end{pmatrix} \begin{pmatrix} x_1(k) \\ x_2(k) \end{pmatrix} + \begin{pmatrix} 0.368 \\ 0.632 \end{pmatrix} u(k)$$

<u>Answer</u>

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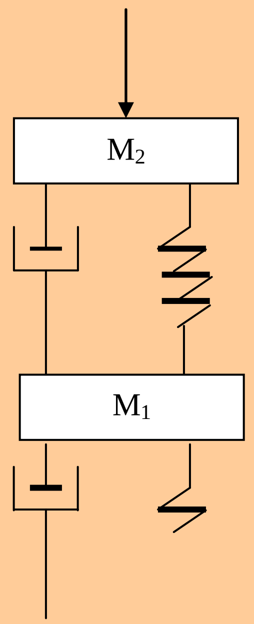
Problem : 7.7 Equivalent electrical network and state equations for a mechanical system

The suspension system for an automobile is designed to increase the comfort of the passengers by absorbing the vibrations due to the terrain of the road. Fig.3 shows the model of such a system. Here M represents the masses, B represents the dashpot damping coefficients, and K represents the spring constants respectively.



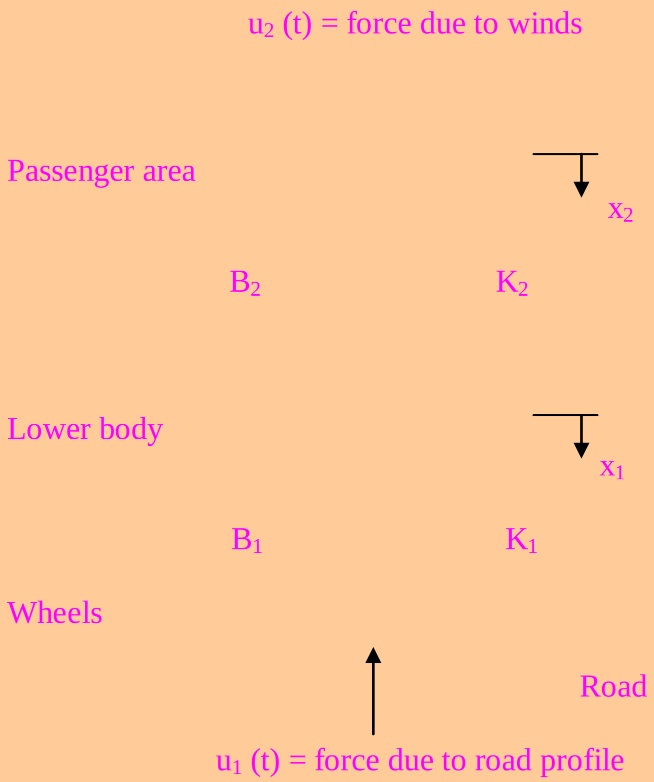
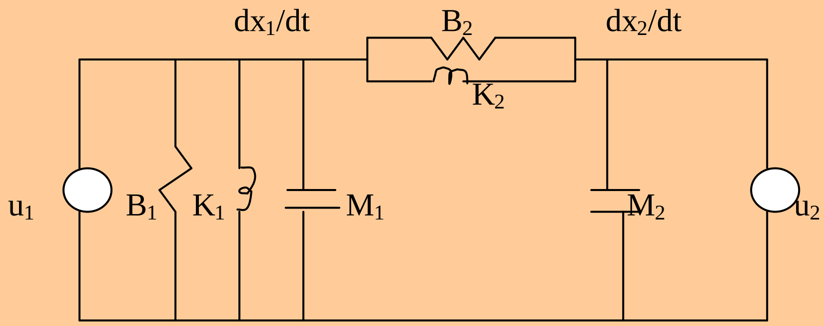


Fig.3

(i) Obtain an equivalent electrical network
(ii) Write a set of state equations for the system
(iii) Describe the procedure for determining the transfer function matrix relating the states x_1 and x_2 to the inputs u_1 and u_2 respectively.

<u>Solution</u>

(a)



(b)

The equilibrium equations are:

$M1\, d^2x_1/dt^2 + B1\, dx_1/dt + K1x1 + B2\, (dx_1/dt - dx_2/dt) + K2\, (x_1 - x_2) = u_1\, (t)$

$M2\, d^2x_2/dt^2 + B2\, (dx_2/dt - dx_1/dt) + K2\, (x_2 - x_1) = u_1\, (t)$

We need four **state variables, x1, dx1/dt, x_2, and dx2/dt. The state eqns. are:**

d**x**/dt=**Ax**+**Bu**

where **A**=
$$\begin{pmatrix} 0 & 1 & 0 & 0 \\ -(K1+K2)/M1 & -(B1+B2)/M & K2/M1 & B2/M1 \\ 0 & 0 & 0 & 1 \\ K2/M2 & B2/M2 & -K2/M2 & -B2/M2 \end{pmatrix}$$

B=
$$\begin{pmatrix} 0 & 0 \\ 1/M1 & 0 \\ 0 & 0 \\ 0 & 1/M2 \end{pmatrix}$$

d**x**/dt =
$$\begin{pmatrix} x1 \\ dx1/dt \\ x2 \\ dx2/dt \end{pmatrix}$$

u =
$$\begin{pmatrix} u1 \\ u2 \end{pmatrix}$$

(c) Transfer function Matrix:
(d)

$$G(s) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ \mathbf{0} & \mathbf{0} & \mathbf{1} & \mathbf{0} \end{pmatrix} \begin{pmatrix} s & -1 & 0 & 0 \\ \mathbf{(K1+K2)/M1} & \mathbf{s+(B1+B2)/M1} & \mathbf{-K2/M1} & \mathbf{-B2/M1} \\ \mathbf{0} & \mathbf{0} & \mathbf{0} & \mathbf{-1} \\ -K2/M2 & -B2/M2 & K2/M2 & s+B2/M2 \end{pmatrix}^{\mathbf{-1}} \begin{pmatrix} 0 & 0 \\ \mathbf{I/M1} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \\ 0 & 1/M2 \end{pmatrix}$$

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Problem : 7.8 Controllability, observability, and stability from the state equations

The state equations of a system are as follows:

$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} 0 & 0 & -8 \\ 1 & 0 & -14 \\ 0 & 1 & -7 \end{pmatrix}\mathbf{x} + \begin{pmatrix} 2 \\ 1 \\ 0 \end{pmatrix}u$$

$$c = \begin{pmatrix} 0 & 0 & 1 \end{pmatrix}\mathbf{x}$$

Determine if the system is

i. controllable
ii. observable, and
iii. stable

Solution

(i) Condition of controllability: Rank of U= (B, AB, A2B) = 3

Condition of controllability: Rank of U = (B, AB, A2B) = 2

$$U = \begin{pmatrix} 2 & 0 & -8 \\ 1 & 2 & -14 \\ 0 & 1 & -5 \end{pmatrix}$$

Rank U = 2, **Uncontrollable**

(ii) Condition of observability: V is non-singular, and Rank of V=(C, CA, CA2)=3

$$V = \begin{pmatrix} 0 & 0 & 1 \\ 0 & 1 & -7 \\ 1 & -7 & 35 \end{pmatrix}$$

V is non –singular, Rank V =3, **Observable**

(ii) Characteristic eqn: $s^3 + 7s^2 + 14s + 8$

S^3	1	14
S^2	7	8
s	90/7	
S^0	8	

Stable

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Problem : 7.9 State equations of the system from the state diagram

(a) Fig.4 shows a feedback control system. The open –loop transfer function is

$$G(s) = \frac{2(s+1)(s+3)}{s(s+2)(s+4)}$$

What is the closed-loop transfer function of the system?

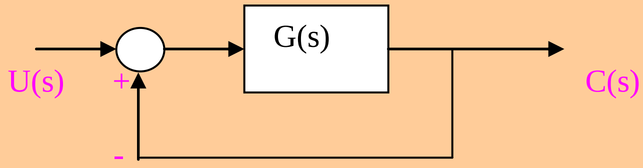

Fig.4

(b) Fig.5 shows the state diagram of the system of Part (a), The state variables are used to provide the output signal.

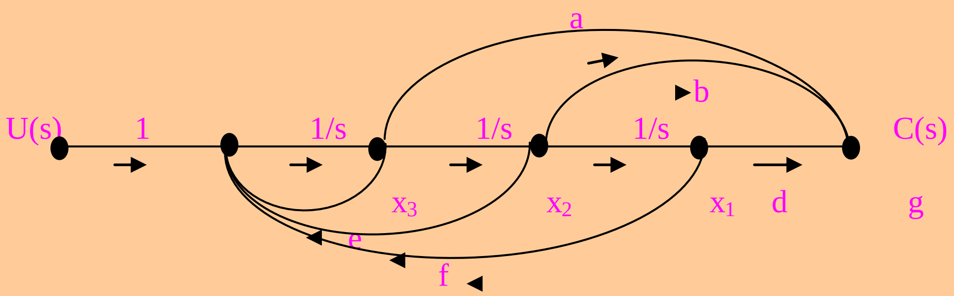

Fig.5

Write the state variable equations of the system in the form:

$$\frac{d\mathbf{x}(t)}{dt} = \mathbf{A}\,\mathbf{x}(t) + \mathbf{B}u(t)$$

$$c(t) = \mathbf{D}\,\mathbf{x}(t)$$

What are the values of transmittances a, b, d, e, f and g shown in Fig.3?

Solution

Closed loop transfer function is

$T(s) = (2s^2+8s+6)/(s^3+8s^2+16s+6)$

The vector differential equation for the state diagram is

$$D\mathbf{x}/dt = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -6 & -16 & -8 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} u(t)$$

and the output is

$$c(t) = (6, 8, 2)\begin{bmatrix} x1 \\ x_2 \\ x_3 \end{bmatrix}$$

a = 2, b = 8, d = 6, e = -8, f = -16, g = -6

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Problem : 7.10 State diagram of the system from the state equations

(a) The state equation of a linear dynamic system is

$$\begin{bmatrix} dx_1/dt \\ dx_2/dt \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} r(t)$$

where x_1 (t), x_2 (t) are the system states and r (t) is the reference input. Determine the system states for t ≥0 when the input r (t) =1 for t ≥0; that is,
r (t) =u (t). The initial state of the system is

$$\begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

(b) Draw the state diagram for the system described in Part (a) by its state equation and initial condition. Assign the state variables as the outputs of the integrators and the reference input and the initial conditions as the system inputs.

Solution

(a)

$$A = \begin{bmatrix} 0 & 1 \\ -2 & -3 \end{bmatrix} \quad B = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$sI-A = \begin{bmatrix} s & -1 \\ 2 & s+3 \end{bmatrix}$$

$$(sI-A)^{-1} = (1/(s^2+3s+2)) \begin{bmatrix} s+3 & 1 \\ -2 & s \end{bmatrix}$$

$$\phi(t) = L^{-1}[(sI-A)^{-1}] = \begin{bmatrix} 2e^{-t}-e^{-2t} & e^{-t}-e^{-2t} \\ -2e^{-t}+2e^{-2t} & -e^{-t}+2e^{-2t} \end{bmatrix}$$

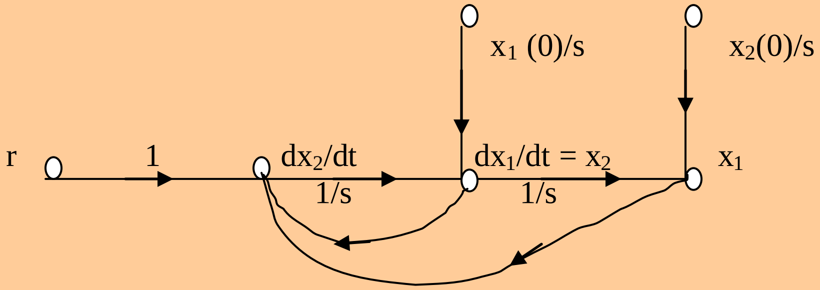
$$x(t) = \boldsymbol{\phi(t)\, x(0)} + \int_0^t \boldsymbol{\phi(t-\tau)\, Br(\tau)\, d\tau}$$

x (t) =

$$\begin{pmatrix} 2e^{-t}-e^{-2t} & e^{-t}-e^{-2t} \\ -2e^{-t}+2e^{-2t} & -e^{-t}+2e^{-2t} \end{pmatrix} \mathbf{x}(0) + \begin{pmatrix} 0.5 - e^{-t} - 0.5e^{-2t} \\ e^{-t} - e^{-2t} \end{pmatrix}$$

$$= \begin{pmatrix} 0.5 + 2e^{-t} - 2.5\, e^{-2t} \\ -2e^{-t} + 3\, e^{-2t} \end{pmatrix}$$

(b)



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Problem : 7.11 Analogue simulation of the system from the transfer function

(a) A robot-arm drive system for one joint can be represented by the differential equation:

$dv(t)/dt = -k_1 v(t) - k_2 y(t) + k_3 i(t)$

where v (t) =velocity, y (t) =position, and i (t) is the control-motor current. Write the second-order drive system in the state-variable form, when $k_1 = k_2 = k_3 = 1$.

(b) Draw the block diagram for analogue computer simulation of a system having the following transfer function:

$Y(s)/U(s) = 1/(s^3+7s^2+14s+8)$.

Choose the state variables as follows:

$x_1 = y(t)$, $x_2 = dx_1/dt$, $x_3 = dx_2/dt$.

Solution

(a) Let x1 = y, x2= dy/dt=v, u= i. Then

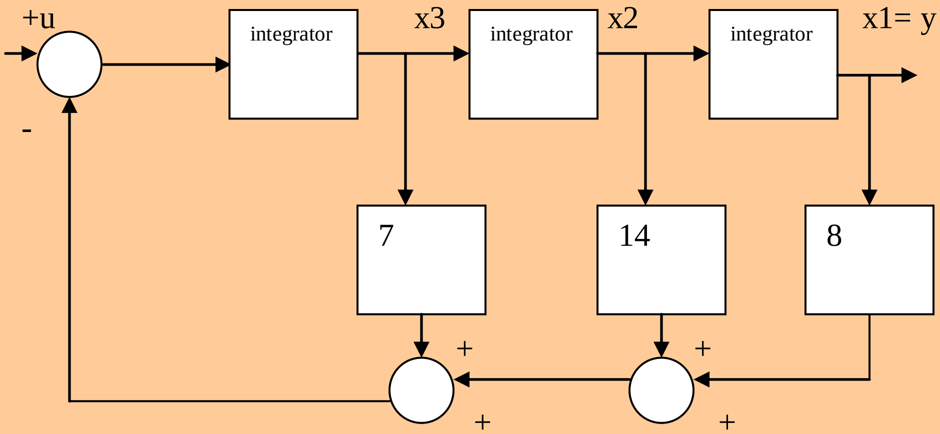
dx1/dt = x2

dx2/dt = -k1x2 -k2x1 +k3 u = -x2 -x1 +u

(b) The transfer function corresponds to the differential equation:

$d^3y/dt^2 + 7d^2y/dt^2 + 14dy/dt + 8y = u$ which can be written as:
$dx_3/dt = -7x_3 - 14x_2 - 8x_1 + u$ and combining this with the state variable description, we get
$dx_1/dt = x_2$
$dx_2/dt = x_3$
$y = x_1$

The state equations lead to the following simple analogue computer simulation.



TOP

Problem : 7.12 Damping ratio of the system by minimizing the quadratic performance index

Determine the value of the damping ratio $\xi > 0$ so that when the system shown in Fig.6 is subjected to a unit step input, the following performance index is minimized:

$$J = \int_0^\infty \mathbf{x}^T(t)\,\mathbf{Q}\,\mathbf{x}(t)\,.dt$$

where

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} e \\ de/dt \end{pmatrix} , \qquad \mathbf{Q} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

Assume the system to be at rest initially.

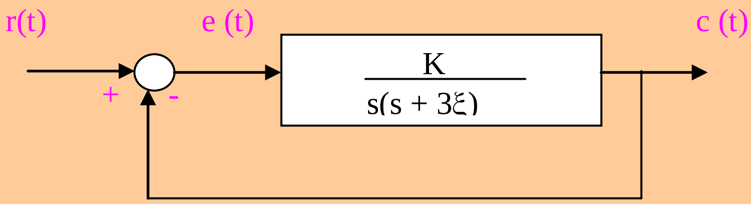


Fig.6

Solution

Eqn. for the system:

$d^2c/dt^2 + 3\zeta dc/dt + c = r$

Since e= r-c, and initial conditions are zero, we have

$d^2x/dt^2 + 3\zeta\, dx/dt + x = r$

From the state –space representation of this eqn. we get

$$A = \begin{pmatrix} 0 & 1 \\ -1 & -3\zeta \end{pmatrix}$$

Note that, since e= r-c, and applied input is a step function, the initial conditions for the state variables are:

$x1\ (0) = 1$
$x2\ (0) = 0$

The performance index is:

$J = x^T (0)\ P\ x\ (0)$

P is determined from

$A^T P + PA = -Q$

(Elements of symmetric P matrix are p11, p12, and p22)
Solving the above eqn. we get

p11 = (1/2ζ) +3ζ/2

p12 = ½

p22 = 1/2ζ

Therefore J = (1/2ζ)+3ζ/2

To minimize J, obtain dJ/dζ = 0. This gives **ζ = 0.577**

TOP

Problem : 7.13 Transfer function of the system from the state equations

(a) An integrator whose transfer function is 1/s has a negative feedback path with an adjustable gain K.

(i) Demonstrate the manner in which the gain K affects the closed-loop transfer function and the response of the system, when it is varied from zero to plus and minus infinity.

(ii) Draw the locus of the closed loop pole in the complex s- plane.

(b) Obtain the transfer function Y(s)/U(s)of the system shown in Fig.7 where x_1 and x_2 are the state variables.

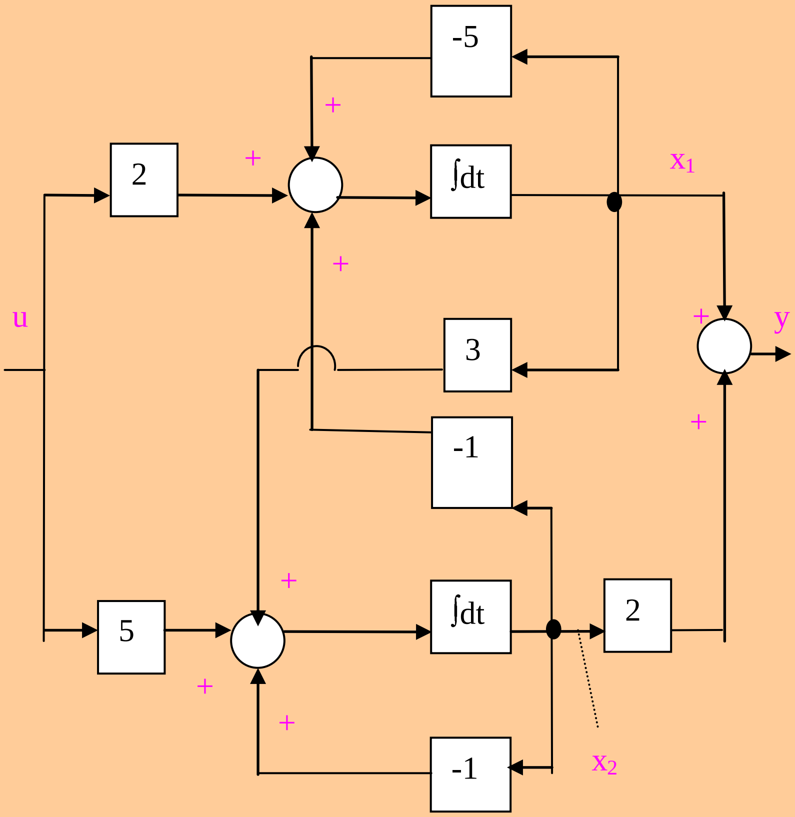
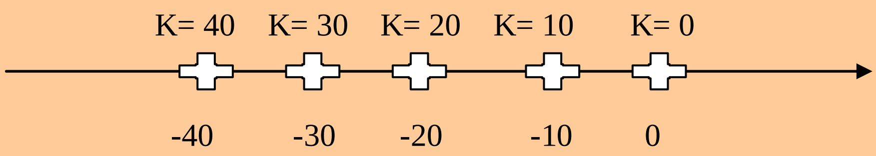


Fig.7

<u>Solution</u>

(a)

The closed loop transfer function is T(s) =1/(s+K). Pole is at s = -K. Addition of the feedback path has caused the transfer function pole to shift from the origin to the left in the s-plane as K is increased from some positive value to . While increasing K, the system has been changed from an integrator to a first-order low –pass filter whose half-power bandwidth is K rad/sec. For negative K , system will be unstable. Locus of closed loop poles in the complex plane as K is varied is sown below.



(b)

Write dx1/dt = -5x1 — x2 + 2u

dx2/dt = 3x1-x2 + 5u

y = x1+ 2x2

Transfer function $G(s) = C(sI-A)^{-1}B$

$$= [1 \;\; 2] \begin{pmatrix} s+5 & 1 \\ -3 & s+1 \end{pmatrix}^{-1} \begin{pmatrix} 2 \\ 5 \end{pmatrix} = (12s+59)/(s+2)(s+4)$$

TOP

Problem : 7.14 Solving the state equation

(a) Show how to reduce the following differential equation

.

$$d^3x/dt^3 + a_1.d^2x/dt^2 + a_2 .dx/dt + a_3 .x = u$$

into the following set of first-order differential equations:

$$\begin{pmatrix} dx_1/dt \\ dx_2/dt \\ dx_3/dt \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -a_3 & -a_2 & -a_1 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad u$$

(b) A system is defined by the state equations:

$dx_1/dt = -2x_1 + x_2$

$dx_2/dt = x_1 - 2x_2 + u$

and the output equations:

$y_1 = 3x_1 + x_2$

$y_2 = x_1 + 2x_2$

$y_3 = x_2$

The initial conditions are:

$x_1(0) = 1$, $x_2(0) = 2$.

Find the time solution of the output variables when the input u =1 for t >0 and 0 for t < 0.

Solution

(a)

$d^3x/dt^3 + a_1.d^2x/dt^2 + a_2 .dx/dt + a_3 .x = u$

Choose x1= x, x2 = dx/dt, x3 = d^2x/dt^2 so that dx3 /dt = d^3x/dt^3

Then the differential equation may be written as

dx1/dt = x2
dx2/dt = x3
dx3/dt =- a3x1- a2x2- a1x3 + u.

These first order equations are then written in the matrix-vector form.

(b)

If a = e^{-t} and b = e^{-3t}, then the solution for the states is

X1 = (1/3) +a – (b/3)

X2=(2/3) +a + (b/3) and the outputs are:

Y1=(5/3) +4a – (2b/3)

Y2=(5/3) +3a + (b/3)

Y3=(2/3) +a + (b/3)

TOP

Problem : 7.15 Design of a state variable feedback controller

(a) Describe a system involving feedback that arises in the following items:

i. Automobiles
ii. Economics

Attempt to identify the plant or process, the input (both reference and disturbance), and the feedback path(s).

(b) A multi-input /multi-output plant with a state-variable feedback controller is shown in Fig.8.

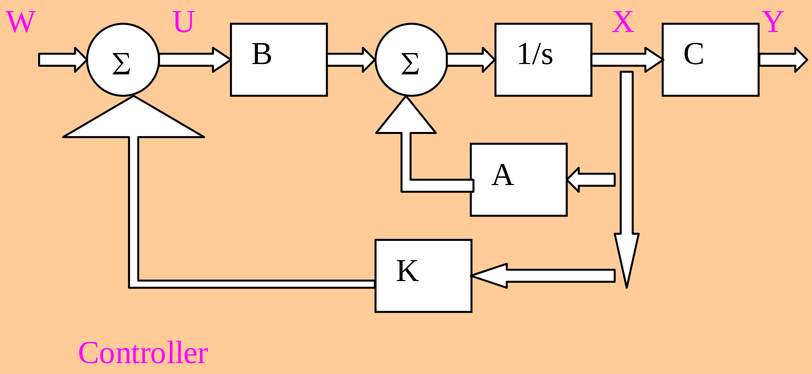


Fig.8

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & -30 & -11 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 1 & 0 & 0 \end{pmatrix}$$

Find the feedback matrix K such that the poles are at s= -2,-5, -6.

Solution

(a)

Category	Feedback system	Plant	Reference input	Disturbance input	Feedback path
Economics	Supply of goods	Production & distribution	Desired profit	Cost of raw material	Prices
Automobile	Automatic choke	Carburetor	Fuel/air ratio versus engine temperature	Engine temperature	Heat sensor on choke

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TOP

Problem : 7.16 State variable model of a hoist

A hoist may be thought of as a motor lifting a mass in the presence of gravity, as shown in Fig.9

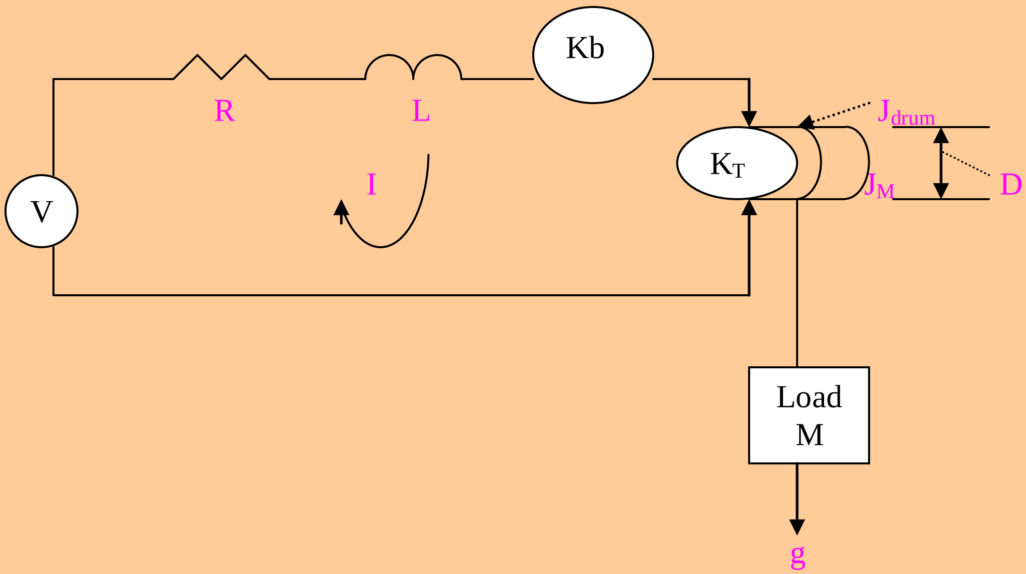


Fig.9

Write the appropriate equations and develop a state variable model. Kb is the back emf constant and K_T is the torque constant. J_M and J_{drum} are the moments of inertia of the motor and drum respectively.

Solution

V= IR +LdI/dt +Kb dc/dt

K_T I =(J_M + J_{drum}) d^2c/dt^2 +MgD/2
Let J= J_M +J_{drum}

Choose x1=c,x2= dc/dt, x3= I.

Substitute & manipulate:

$$\frac{d\mathbf{x}}{dt} = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & K_T/J \\ 0 & -Kb/L & -R/L \end{pmatrix} \begin{pmatrix} x1 \\ x2 \\ x3 \end{pmatrix} - \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} MgD/2J + \begin{pmatrix} 0 \\ 0 \\ 1/L \end{pmatrix} V$$

TOP

Problem : 7.17 Design of the observer for given observer poles

(a) State the conditions for complete observability in terms of transfer functions or transfer matrices.

(b)For a unity feedback closed loop system having an open loop transfer function

$$G(s) = 100/(s(s^2 +0.1s +10)),$$

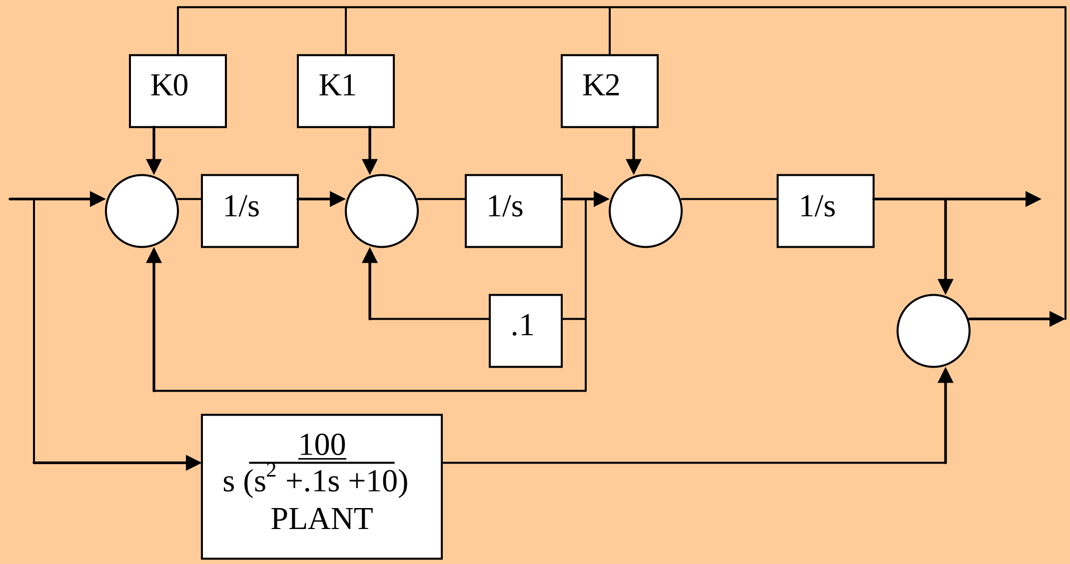
write the state equations and design an observer for observer poles at - 50, -10-j10, -10+ j10.

Solution

dx1/dt = x2

dx2/dt = 1x2+x3

dx3/dt = 10x2 +100 (R-x1)



Observer characteristic equation:

$s^3 + (.1 +10K2)s^2 +(10 +100K1 +K2)s+100K0 +100K2=0$

For observer poles s=-50,-10+ j10,-10-j10,

$s^3 +70s^2+1200s +10000=0$

Equating coefficients, K2=6.99, K1=19.83, K0=80.17

TOP

Problem : 7.18 State equations of a d.c shunt motor

A d.c shunt motor, Fig.10, drives an inertia load. The shaft connecting motor to loads is not rigid and must be considered as spring. The applied voltage is V.

R=armature resistance
L=armature inductance
K_b=back emf constant
K_T= Torque constant
K_s=spring constant
J_M=rotor moment of inertia
J_L=load moment of inertia
I =armature current
θ_M=rotor angle
θ_L=load angle, Write the state equations of the control system using θ_M, θ_L, I, $d\theta_M/dt$, $d\theta_L/dt$ as the state variables.

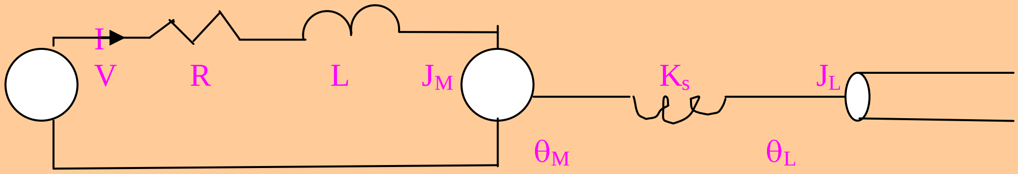


FIG. 10

Solution

$V= IR +L\, dI/dt + K_b\, d\theta_M/dt$

$K_T I = J_M\, d\theta_M/dt + K_s\ (\theta_M - \theta_L)$

$0= J_L\, d\theta_L/dt - K_s\ (\theta_M - \theta_L)$

$x_1 = \theta_L$

$x_2 = d\theta_L/dt$

$x_3 = \theta_M$

$x_4 = d\theta_M/dt$

$x_5 = I$

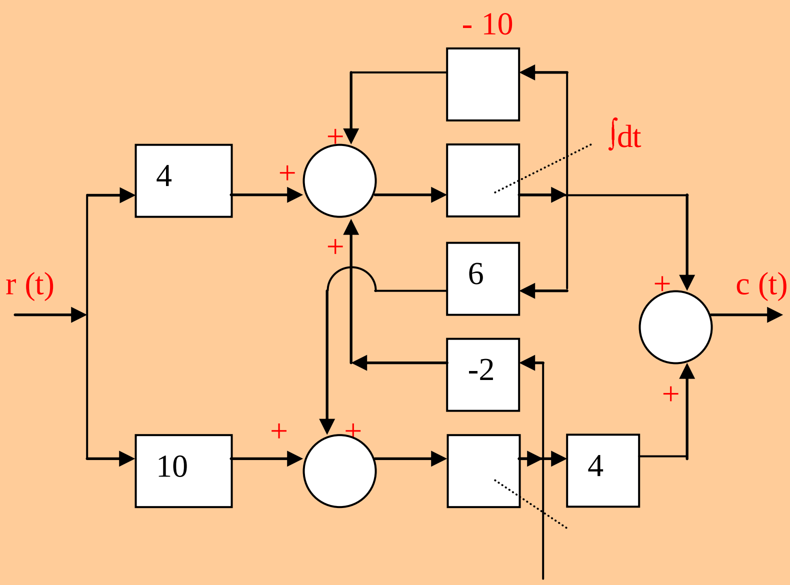
$$A = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -K_s/J_L & K_s/J_L & 0 \\ 0 & 0 & 0 & 1 & 0 \\ K_s/J_M & 0 & K_s/J_M & 0 & K_T/J_M \\ 0 & 0 & 0 & K_b/L & R/L \end{pmatrix}$$

$B^T = (0 \quad 0 \quad 0 \quad 0 \quad K_b/L)$

TOP

Problem : 7.19 Transfer function of the system from the state diagram

(a) Write down the state and output equations of the system shown in Fig.11.
(b) Obtain its transfer function, C(s)/R(s)



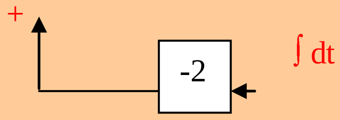


Fig.11

Solution

:

$dX_1/dt=-10x_1-2x_2+44$

$dX_2/dt=-6x_1-2x_2+104$

$y=x_1+4\,x_2$

$A=\begin{pmatrix} -10, & -2 \\ 6, & -2 \end{pmatrix}$ $B=\begin{pmatrix} 4 \\ 10 \end{pmatrix}$ $C=[\ 1,4\ ]$

Transfer function = $C\,(sI-A)^{-1}B = 11(s+11)/[(s+1)(s+2)]$

TOP

Problem : 7.20 Design of a state variable feedback controller

(a) Discuss the realization of a feedback controller for pole allocation.
(b) Fig.12 shows a single-input single-output second-order system with state-variable feedback controller. The system equations are:
(c)

$d\mathbf{x}/dt = \mathbf{A}\,\mathbf{x} + \mathbf{b}\,u$

$y = \mathbf{C}^t\,\mathbf{x}$

where

$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}$, $\mathbf{b} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$,

$\mathbf{C}^t = [1 \quad 0]$

The plant input, u and the system input, w are related by

$$U = w + \mathbf{Kx}\ .$$

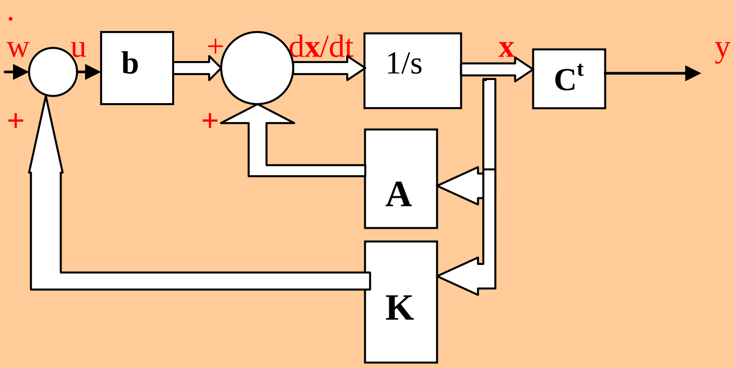


Fig.12

Find the feedback matrix **K** such that the poles are at

s = -1,-2.

Solution

$$A=\begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix} \quad B=\begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$C=\begin{pmatrix} 1 & 0 \end{pmatrix}$$

T.F = Y(s)/U(s) = $C(sI-A)^{-1}$ B= 1/(s-1)(s-2) with poles at 1, 2.

Desired characteristic equation (s+1) (s+2)= $s^2+3s+2=0$

Feedback matrix K= (K1,K2)

$$A_{new} = A + BK = \begin{pmatrix} 1 & 1 \\ 2+K1 & K2 \end{pmatrix}$$

det $(sI - A_{new}) = 0$. This gives s^2-(K2+1) s-2K1=0. Compare with desired equation and obtain

K1=0, K2=-4

TOP

Problem : 7.21 Transfer function of the system from the state diagram

(a) Write the state and output equations:

$d\mathbf{x}/dt = \mathbf{Ax}+\mathbf{Bu}$
$\mathbf{y} = \mathbf{Cx}+\mathbf{Du}$

of the system shown in Fig.13.

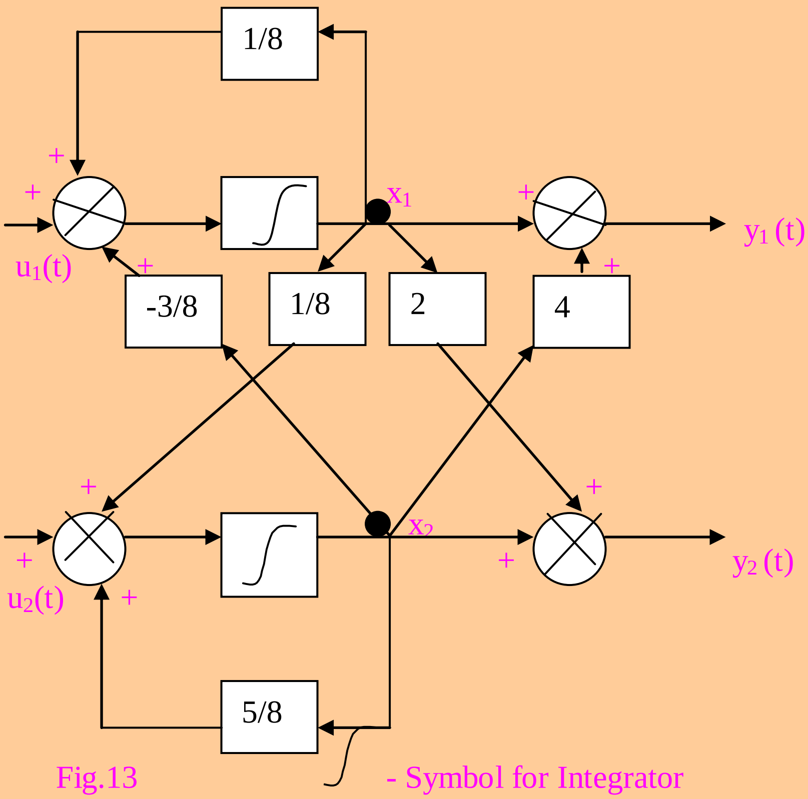


Fig.13

(b) -If the state space description of the system of Fig.1 is given by

$$\mathbf{A} = \begin{pmatrix} 1/8 & -3/8 \\ 1/8 & 5/8 \end{pmatrix} \qquad \mathbf{B} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 1 & 4 \\ 2 & 1 \end{pmatrix}$$

D = a null matrix of proper order , find the transfer function matrix relating the input vector with the output vector.

Solution

(a) $dx_1/dt = x_1/8-3x_2/8 +u_1$

$dx_2/dt = x_1/8+5x_2/8 +u_2$

$y_1= x_1+4x_2$

$y_2= 2x_1+x_2$

(b) B=Unit matrix, D= null matrix

$$G(s) =C(sI-A)^{-1}B+D$$

$$= \begin{pmatrix} 1 & 4 \\ 2 & 1 \end{pmatrix} \left(\begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \begin{pmatrix} 1/8 & -3/8 \\ 1/8 & 5/8 \end{pmatrix} \right)^{-1}$$

$$= \begin{pmatrix} a/e, & b/e \\ c/e, & d/e \end{pmatrix}$$

where $a= 8s-1, b=32s-1, c=16s-9, d=8s-7, e=8s^2-6s+1$

TOP

Problem : 7.22 The state transition matrix

(a) Find the inverse of matrix A.

$$A = \begin{pmatrix} 5 & 0 & 2 \\ 0 & 2 & -3 \\ 2 & 1 & 2 \end{pmatrix}$$

(b) Find the characteristic polynomial and the eigenvalues of matrix B.

$$B = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ -4 & -6 & -4 \end{pmatrix}$$

(c) Find the state-transition matrix of the system described by the following state equations:

$$dx_1/dt = x_2$$
$$dx_2/dt = -2x_1-3x_2$$

Solution

(a)

$$\text{Inverse} = (1/27)\begin{pmatrix} 7 & 2 & -4 \\ -6 & 6 & 15 \\ -4 & -5 & 10 \end{pmatrix}$$

(b) Eigenvalues are : 2, -1+j, -1-j

(c) State Transition matrix: $\begin{pmatrix} 2e^{t}-e^{-2t} & e^{t}-e^{2t} \\ -2e^{t}+2e^{2t} & -e^{-t}+2e^{2t} \end{pmatrix}$

TOP

Objective type questions:

(i) Which one of the following statements is true?

(a) State space analysis is not applicable to linear time invariant systems
(b) Conventional root-locus and frequency response methods are applicable to the design of optimal and adaptive control systems that are mostly time varying and/or nonlinear.
(c) State space analysis is a powerful method for time-varying systems, nonlinear systems, and multiple- input-multiple output systems
(d)Frequency response methods are a powerful method for time-varying systems, nonlinear systems, and multiple- input-multiple output systems

Ans: (c)

(ii) Fig. 6 shows a control system defined by the following equations:

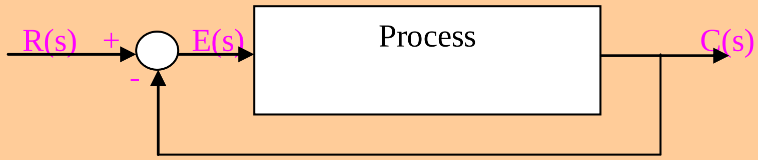


Fig.6

$C(s)/R(s) = 1/[s^2+2\xi s+1]$ $(\xi \geq 1)$

$E(s) = Ł[e(t)] = R(s) - C(s)$

where ξ is the damping ratio. The system is initially at rest and is subjected to a unit –step input. The performance index of the system

$$\int_0^{\infty} t\,|e(t)|\,dt$$

is

(a) 2ξ
(b) $4\xi^2 - 1 + 2\xi$
(c) $4\xi^2 - 1$
(d) ξ

Ans: (c)

(iii) The continuous time system described by

$dx_1/dt = x_1$
$dx_2/dt = -x_1 + u$
$y = x_1$

(a) is completely state controllable but not completely observable
(b) is completely state controllable and completely observable
(c) is completely observable but not completely state controllable

(d)is not completely state controllable and not observable.

Ans: (b)

(iv) For the third order plant which of the following may be a set of state variables?

(a) $x(t)$, $dx(t)/dt$, $d^2x(t)/dt^2$
(b) $x(t)$, $dx(t)/dt$, $x(t) + d^2x(t)/dt^2$
(c) $x(t)$, $d^2x(t)/dt^2$, $x(t) + d^2x(t)/dt^2$
(d) any of the above

Ans: (a)

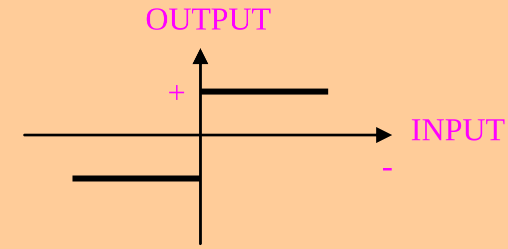
(v) Which one of the following minimum information is necessary to formulate the problem of control systems optimization?

(a) System state equation, output equation and the control vector only
(b) System state equation, output equation, control vector, and the performance index only
(c) System state equation, output equation, control vector, Constraints of the problem, the performance index, and system parameters

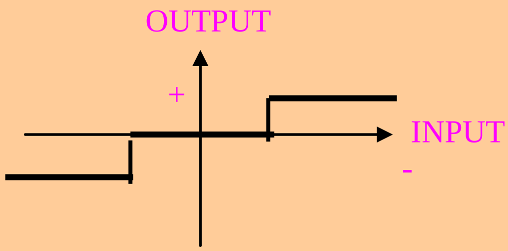
Ans: (c)

(vi) Which one of the following characteristics are the attributes of an ideal relay?

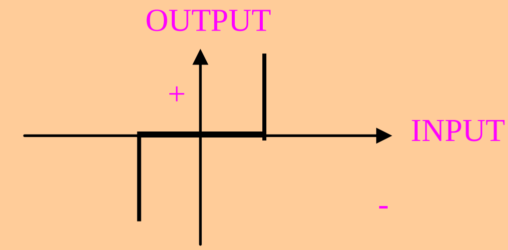
(a)



(b)



(c)



Ans: (a)

(vii) In a series RLC circuit containing a voltage source v (t), the current i and the voltage v_c across the capacitor are chosen as the state variables x_1 and x_2 respectively. Show that the state equations are:

(a) -

$$\begin{bmatrix}\frac{dx_1}{dt}\\[2ex] \frac{dx_2}{dt}\end{bmatrix} = \begin{pmatrix}\frac{R}{L} & \frac{1}{L}\\[2ex] \frac{1}{C} & 0\end{pmatrix}\begin{bmatrix}x_1\\[2ex] x_2\end{bmatrix} + \begin{bmatrix}\frac{1}{L}\\[2ex] 0\end{bmatrix} v\,(t)$$

(b)
$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} \frac{-R}{L} & -\frac{1}{L} \\ \frac{1}{C} & 0 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L} \\ 0 \end{bmatrix} v(t)$$

(c)
$$\begin{pmatrix} \frac{dx_1}{dt} \\ \frac{dx_2}{dt} \end{pmatrix} = \begin{pmatrix} \frac{-R}{L} & -\frac{1}{L} \\ \frac{1}{L} & 0 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{C} \\ 0 \end{bmatrix} v(t)$$

Ans: (b)

(viii) In the RLC circuit of the above question, part (v), show that the characteristic equation of the circuit is

(a) $RCs^2+LCs+1=0$

(b) $s^2+RCs+(1/LC)=0$

(c) $LCs^2+RCs+1=0$

Ans: (c)

(ix)

(a) The following system is *not* completely state controllable:

$$\begin{pmatrix} dx_1/dt \\ dx_2/dt \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \end{bmatrix} u$$

(b) The following system is *not* completely state controllable:

$$\begin{pmatrix} dx_1/dt \\ dx_2/dt \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix} u$$

(c) The following system is *not* completely observable:

$$\begin{pmatrix} dx_1/dt \\ dx_2/dt \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} 0 \\ 1 \end{pmatrix} u$$

$$y = (1 \quad 0) \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Ans: (b)

(x) With the knowledge of the state variable description of a plant, the transfer function of the plant

(a) Cannot be determined
(b) Can be determined completely
(c) There is no relationship between the state variables and the transfer function

Ans: (b)

(xi) Which one of the following statements is true?

(a) A scalar function V (**x**) is said to be *positive definite* in a region **Ω**(which includes the origin of the state space) if V (**x**)< 0 for all nonzero states **x** in the region **Ω** and V (**0**) = 0

(b) A scalar function V (**x**) is said to be *negative definite if minus of* V (**x**) is negative definite

(c)A scalar function V (**x**) is said to be *positive semi definite if it is positive at all states in the region* **Ω** except at the origin and at certain other states, where it is zero

Ans: (c)

(xii) Which of the following is the **A** matrix of the system?

(a) $\begin{pmatrix} 0 & 1 \\ -5 & -4 \end{pmatrix}$

(b) $\begin{pmatrix} 0 & 1 \\ 5 & 4 \end{pmatrix}$

(c) $\begin{pmatrix} -5 & -4 \\ 0 & 1 \end{pmatrix}$

Ans: (a)

(xiii) In a control system

(a) The state variable can *always* be measured
(b) The output variable cannot be defined as a function of the state variables
(c) The output is a variable that can be measured

Ans: (c)

(xiv) The state transition matrix $\phi(t)$ possesses the following properties:

(a) $[\phi(t)]^k = \phi(kt)$ for k = integer
(b) $\phi^{-1}(t)$ is not equal to $\phi(-t)$
(c) $\phi(t_2-t_1)\,\phi(t_1-t_0)$ is not equal to $\phi(t_2-t_0)$ for any t_0, t_1, t_2

Ans: (a)

(xv) Which one of the following statements is true?

(a) The frequency domain techniques are applicable to non-linear and time-varying systems.
(b) The frequency domain techniques are particularly useful for multivariable control systems
(c) Time- domain methods cannot be utilized to design an optimum control system
(d) Time- domain techniques can be utilized for non-linear, time-varying, and multivariable systems.

Ans:(d)

(xvi) The state-variable equations of a single-input single-output system are:

$d\mathbf{x}/dt = \mathbf{A}\mathbf{x} + \mathbf{B}u$
$y = \mathbf{C}^t\mathbf{x}$

where **A** is of order (nxn), and **B** and **C** are each of order (nx1).

Which one of the following describes the transfer function, G(s)= y(s)/u(s), for this system?

(a) $(s\,\mathbf{I}-\mathbf{A})^{-1}\,\mathbf{B}\mathbf{C}^t$
(b) $\mathbf{C}^t\,\mathbf{B}\,(s\mathbf{I}-\mathbf{A})^{-1}$
(c) $\mathbf{C}\,(s\mathbf{I}-\mathbf{A})^{-1}\,\mathbf{B}$

where I is an (nxn) identity matrix.

Ans: (c)

(xvii) The transfer function

(a) can be uniquely determined from the state variable equations and the state variable equations can also be uniquely determined from the transfer function.

(b) for a given set of state variable equations is not unique, but the state variable equations can be uniquely determined from the transfer function.

(c) can be uniquely determined from the state variable equations, but the state variable equations for a given transfer function are not unique.

Ans: (a)

(xviii) Which one of the following statements is not true?

(a) A SISO (single-input single-output) system with the state equations

$dx_1/dt = -3x_1 - x_2 + u$
$dx_2/dt = 2x_1$
$y = x_1 - x_2$

is controllable

(b) A SISO system with the state equations

$dx_1/dt = -3x_1 - x_2 + u$
$dx_2/dt = 2x_1$
$y = x_1 - x_2$

is observable.

(c) A SISO system with the state equations

$dx_1/dt = -2x_1 + 3x_2 + u$
$dx_2/dt = x_1$
$y = x_1 + u$

is unobservable.

Ans: (c)

(xix) For the state-transition equation,

$$\mathbf{x}(t) = \phi(t-t_0)\,\mathbf{x}(t_0)+\int_{t_0}^{t} \phi(t-\tau)\,\mathbf{Br}(\tau)\,d\tau,$$

the Laplace transformed equation is

(a) $\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})\,\mathbf{x}(t_0) + (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{BR}(s)$
(b) $\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{x}(t_0) + (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{BR}(s)$
(c) $\mathbf{X}(s) = (s\mathbf{I} - \mathbf{A})^{-1}\mathbf{x}(t_0) + (s\mathbf{I} - \mathbf{A})\,\mathbf{BR}(s)$

Ans: (b)

(xx) For a third-order plant, which of the following may be a set of state variables?

(a) y, dy/dt, d2y/dt^2
(b) y, dy/dt, y + d^{2}y/dt^2
(c) y, d^{2}y/dt^2, y + d^{2}y/dt^2

Ans: (a)

(xxi) Which one of the following systems is completely state controllable?

(a) $dx_1/dt = x_2$
$dx_2/dt = -x_1+u$
(b) $dx_1/dt = -2x_1+x_2+u$
$dx_2/dt = -x_2$
(c) $dx_1/dt = -0.5x_1$
$dx_2/dt = -2x_2+u$
(d) $dx_1/dt = -x_1+2u$
$dx_2/dt = -2x_2$

Ans: (a)

(xxii) Which one of the following systems is completely observable?

(a) $dx_1/dt = -x_1$
$dx_2/dt = -2x_2$
$y = x_2$
(b) $dx_1/dt = x_1+x_2$
$dx_2/dt = -2x_1-x_2+u$
$y = x_1$

(c) $dx_1/dt = -2x_1+3u$
$dx_2/dt = -x_2+u$
$y = x_1$
(d) $dx_1/dt = x_2$
$dx_2/dt = -x_1-2x_2$
$y = x_1$

Ans: (b)

(xxiii) The transfer function

(a) can be uniquely determined from the state variable equations and the state variable equations can also be uniquely determined from the transfer function.
(b) for a given set of state variable equations is not unique, but the state variable equations can be uniquely determined from the transfer function.
(c) can be uniquely determined from the state variable equations, but the state variable equations for a given transfer function are not unique.
(d) cannot be uniquely determined from the state variable equations, and the state variable equations cannot be uniquely determined from the transfer function.

Ans:(c)

(xxiv) The state transition matrix of the system defined by

$dx_1/dt = x_1$
$dx_2/dt = -2x_1-3x_2$

is

(a) $\begin{pmatrix} 2a & -b \\ 2b & -a \end{pmatrix}$

(b) $\begin{pmatrix} 2a-b & a-b \\ -2a+2b & -a + 2b \end{pmatrix}$

(c) $\begin{pmatrix} a-b & a-b \\ -a+b & -a + b \end{pmatrix}$

(d) $\begin{pmatrix} 2a-b & a-b \\ -2a-2b & -a - 2b \end{pmatrix}$ where $a = e^{-t}$, $b = e^{-2t}$

Ans: b

(xxv) The number of rabbits is x_1 , and if left alone would grow indefinitely until the food supply was exhausted.

$dx_1/dt = Kx_1$

However, with foxes present, we have

$dx_1/dt = Kx_1 - a\,.x_2$

where x_2 is the number of foxes. Now, if the foxes must have rabbits to exist, we have

$dx_2/dt = -hx_2 + b\ .x_1$

This system is stable and thus decays to the condition

$x_1(t) = x_2(t) = 0$ at $t = \infty$ when

(a) h >K, ab > Kh
(b) h <K, ab < Kh
(c) h>K, ab < Kh

Ans:a

(xxvi) In the problem of rabbits and foxes given in Part (xv), if K=1, h=3, and a=b=2, then

the transition matrix of the system is

(a) $$\Phi(t) = \begin{pmatrix} 2e^{-t} & -2\,e^{-t} \\ 2\,e^{-t} & -2t\,e^{-t} \end{pmatrix}$$

b) $$\Phi(t) = \begin{pmatrix} (2t+1)e^{-t} & -2t\,e^{-t} \\ 2t\,e^{-t} & (-2t+1)\,e^{-t} \end{pmatrix}$$

c) None of the above.

Ans :b

(xxvii) In a series RLC circuit, if the current in the circuit and the voltage across the capacitor are selected as the state variables, x_1 and x_2, respectively, then the state space description of the circuit is given by

(a)

$$\mathbf{A} = \begin{pmatrix} -R/L & -1/L \\ 0 & 1/C \end{pmatrix} \quad B = \begin{pmatrix} 1/L \\ 0 \end{pmatrix}$$

(b)

$$\mathbf{A} = \begin{pmatrix} -R/L & -1/L \\ 1/C & 0 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 1/C \\ 0 \end{pmatrix}$$

(c)

$$\mathbf{A} = \begin{pmatrix} -R/L & -1/L \\ 1/C & 0 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 1/L \\ 0 \end{pmatrix}$$

Ans: c

(xxviii) Which one of the following statements is <u>not</u> true?

(a) A SISO(single-input single-output) system with the state equations

$dx_1/dt = -3x_1 - x_2 + u$
$dx_2/dt = 2x_1$
$y = x_1 - x_2$

is controllable.

(b) A SISO system with the state equations

$dx_1/dt = -2x_1 + 3x_2 + u$
$dx_2/dt = x_1 + u$
$y = x_1$

is uncontrollable.

(c) A SISO system with the state equations

$dx_1/dt = -3x_1 - x_2 + u$
$dx_2/dt = 2x_1$
$y = x_1 - x_2$

is observable.

(d) A SISO system with the state equations

$dx_1/dt = -2x_1 + 3x_2 + u$
$dx_2/dt = x_1$
$y = x_1 + u$

is unobservable.

Ans: (d)

(xxix) Which of the systems with the following state space descriptions is completely controllable?

(a)

$$\mathbf{A} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 2 \\ 5 \end{pmatrix}$$

(b)

$$\mathbf{A} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

(c)

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 2.5 & -1.5 \end{pmatrix} \quad \mathbf{B} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

Ans:a

(xxx) Which of the following systems is <u>not</u> completely observable?
Note that $\mathbf{C}^t$ is the transpose of **C.**

(a)

$$\mathbf{A} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} \quad \mathbf{C}^t = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

(b)

$$\mathbf{A} = \begin{pmatrix} -1 & 0 \\ 0 & -2 \end{pmatrix} \quad \mathbf{C}^t = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

(c)

$$\mathbf{A} = \begin{pmatrix} 1 & 1 \\ -2 & -1 \end{pmatrix} \quad \mathbf{C}^t = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

Ans: b

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