Performance of Feedback Control Systems

Design of a PID Controller	Transient Response of a Closed Loop System	Damping Coefficient, Natural frequency, and Settling time	Steady-state Error and Type 0, Type 1 , and Type 2 Systems	Steady-state error of feedback control systems
Gain and parameters of a system for specified settling time and overshoot in the step response.	Time domain performance specifications			**Objective type questions:**

WEBSITE takes you to start page after you have read this chapter.
Start page has links to other chapters.

Problem 2.1 Design of a PID Controller

The linearized transfer function of a helicopter near hover is

$$G(s) = K(s+0.3)/ [(s+0.65) (s^2-0.2s +0.1)].$$

Design a PID controller to stabilize the system as well as to reduce the steady-state error to step inputs to zero. The controller is in cascade with G(s) in a unity feedback system. The transfer function of the PID controller is selected as

$$G_c(s) = [s + 0.5)(s + 1)]/s$$

Solution:

The unity feedback system with forward path transfer function is

$$G_c(s)G(s) =[K(s+0.3)(s+0.5)(s+1)]/[s(s+0.65)(s^2-0.2s+0.1)]$$

Using the root locus method, K required to obtain closed loop poles with damping ratio 0.707 is K=2.748.

For this value of K , the poles of the close loop transfer function are located at -0.256, -0.574, and $-1.184 \pm j\, 1.184$

Response of closed loop system to unit step is

C(t) = 1- 0.1187exp(-0.256t)+0.0733exp(-0.574t)-0.555 exp(-1.184t) cos (1.185t) +1.377 exp(-1.184t) sin (1.185t).

The steady-state error to a unit ramp input = $1/K_v$ = 0.65x 0.1/(2.748x 0.3x 0.5x 1) = 0.158

TOP

Problem 2.2 Transient Response of a Closed Loop System

A second –order servo system has unity feedback and an open-loop transfer function

$$G(s) = 100/[s(s+10)]$$

(a) What is the characteristic equation of the closed loop system?
(b) What are the numerical values of the damping factor, ξ and the natural frequency, ω_n?
(c) Derive or write down the expression of the transient response of the system to a unit step input in terms of ξ and ω_n?
(d) Estimate the time from the start of the transient to the maximum overshoot
(e) What is the settling time of the system?
(f) What is the steady-state error for a ramp input of 0.5 rad/s?

Solution:

(a) $s^2+10s+100=0$

(b) $\omega_n=10$, $\xi=0.5$

(c) $C(s) = w_n^2 R(s)/[s^2+2\xi w_n s + w_n^2]$

$c(t) = 1- [e^{-\xi wnt}\sin(w_n\beta t+\theta)]/\beta$

where $\beta = \sqrt{(1-\xi^2)}$ and $\theta = \tan^{-1}\beta/\xi$

(d) $T_p = \pi/w_n\sqrt{(1-\xi^2)} = \pi/10\sqrt{0.75} = 0.362$ s

(e) $Ts = 4/\xi w_n = 4/0.5\text{x}10 = 0.8$ s

(f) $e_{ss} = R/K_v = .05/100/10 = 5\text{ x}10^{-5}$

[TOP]

Problem 2.3 Damping Coefficient, Natural frequency, and Settling time

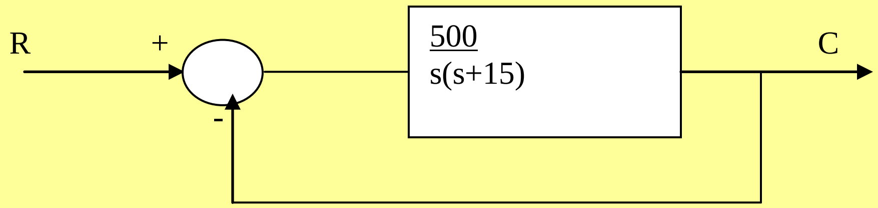
A second-order servo has unity feedback and an open-loop transfer function:

$$G(s) = \frac{500}{s(s+15)}$$

i. Draw a block diagram for the closed –loop system
ii. What is the characteristic equation of the closed loop?
iii. What are the numerical values of the damping coefficient, ξ and the natural frequency, ω_n?
iv. Sketch and label a typical transient response to a unit step input.
v. Calculate the time from the start of transient to maximum overshoot.
vi. What is the settling time of the system?
vii. If the system is subjected to a ramp input of 0.5 rad/s, what is the steady-state error? ,

Solution:

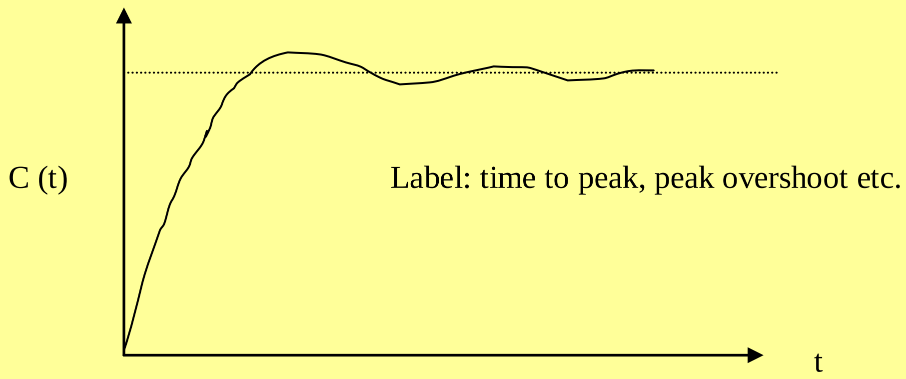
(i)



(ii) $s^2+15\,s+500=0$

(iii) $\omega_n=\sqrt{500}$, $\xi=15/(2\sqrt{500})$

(iv)



(v) Time to peak =0.15 sec
(vi) Settling time= 0.53 sec
(vii) E_{ss}= 0.5/(500/15)

TOP

Problem 2.4 Steady-state Error and Type 0, Type 1 , and Type 2 Systems

(a) Explain the meanings of the terms: Type 0, Type 1, and Type 2 control systems.

(b) Fig.1 shows the steering control system of a mobile robot.

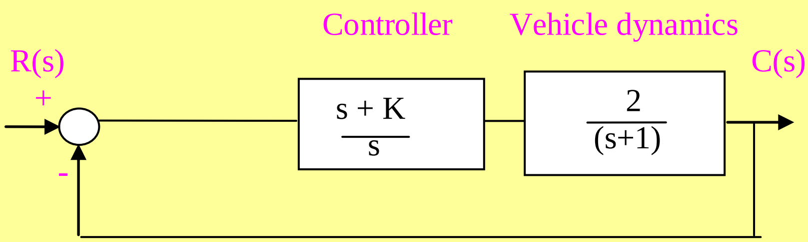


Fig.1

(i) Find the steady-state error of the system for a step input when K =0 and when K = 1.

(ii) Find the steady-state error of the system for a ramp input when K = 1.

(iii) Determine and sketch the transient response of the system to a step input when K = 1.

Solution:

(b)
G1(s) = (s+K)/s
G(s)= 2/(s+1)
(i) 1/(1+2K) = 1/(1+2*1)=1/3
(ii)Zero
(i) 1/2K =1/2

(c)

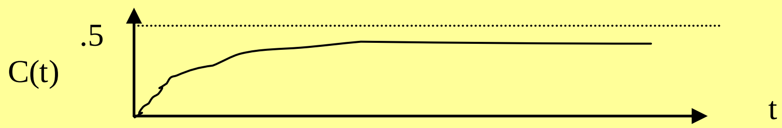
$G1(s) = (s+1)/s$

$G(s) = 2/(s+1)$

$T(s) = 1/(1+G(s)) = s/(s+2)$

$C(s) = T(s).R(s) = s/(s+2)*(1/s^2)$

$C(t) = 0.5(1- \exp(-t))$



<u>Problem 2.5</u>   Steady-state error of feedback control systems

(a) The input to a linear control system with unity feedback is

$$r(t)=r_o +r_1.t+(r_2/2).t^2$$

Find the steady-state error in the output of the system in terms of K_p, K_v and K_a ,where

K_p= position error constant

K_v = velocity error constant

K_a = acceleration constant.

When does the error become infinity?

(b) Compare the errors of the unity feedback control systems with the following open-loop transfer functions.

$G_1(s) = 10/[s(s+1)]$ ________

$G_2(s) = 10/[(s(2s+1)]$ ________

-The input in both cases is

$$r(t)=5 +2t+3t^2$$

Solution:

(a)

e_{ss} =(r_o/(1+Kp)) +(r_1/Kv) +(r_2/Ka)

steady-state error becomes infinity unless the system is of Type 2 or higher.

(b)

E1(s)/R(s) = 1/(1+G1(s)) = $(s+s^2)/(10+ s+s^2)=.1s+.09s^2- .019s^3+\ldots$

E2(s)/R(s) = 1/(1+G2(s)) = $(s+2s^2)/(10+ s+2s^2)=.1s+.19s^2- .039s^3+\ldots$

Error time functions are:

$$\lim_{t \to \text{infinity}} e1(t) = 0.1(2+6t)+.09*6$$

$$\lim_{t \to \text{infinity}} e2(t) = 0.1(2+6t)+.19*6$$

Lim e1(t) < Lim e2(t)

TOP

Problem 2.6 Gain and parameters of a system for specified settling time and overshoot in the step response.

(a) Fig. 2 shows a feedback control system. Find the values of gain K and parameter p so that a reasonably fast step response with a settling time less than four seconds and an overshoot of less than 5% is achieved.

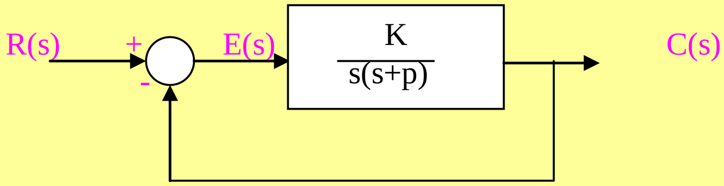


Fig.2

(b) Fig. 3 shows a control system that reduces the effect of rapid scanning motion in a TV camera system. This is the problem of jumping or wobbling of the TV picture when the camera is mounted in a moving truck.

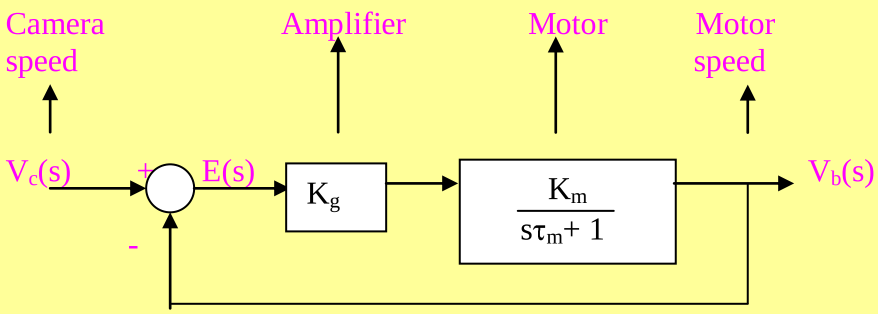


Fig.3

A maximum scanning motion of 25^{o} per second is expected.

i. Determine the error of the control system E(s).
ii. Determine the loop gain K_gK_m for a permissible steady-state error of 1^{o} per second.

Solution:

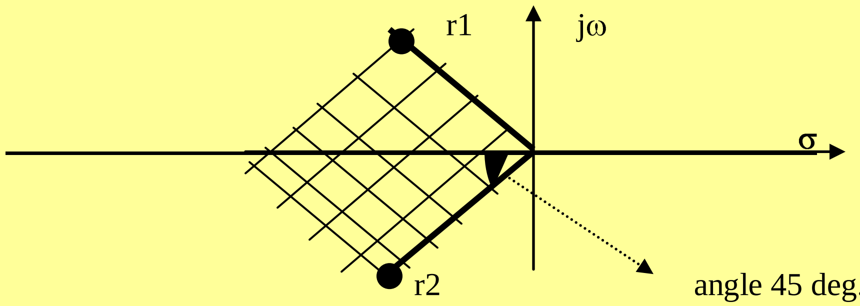
(a)

$T_s = 4/\xi\omega_n$(Response remains within 2% after 4 time constants)
$T_s = 4/\xi\omega_n < 4$ sec.

We require real part of complex poles of CLTF, T(s) to be

$\xi\omega_n >$ or $= 1$

Region satisfying both time domain requirements is shown cross-hatched in Fig. below



$\xi > .707$, $\xi\omega_n >$ or $= 1$

Closed loop roots are chosen ,as the limiting point in order to provide the fastest response , as

r1 =-1+j1
r2 =-1-j1

$\xi=1/\mathrm{sqrt}(2)$, $\omega_n=1/\xi=\mathrm{sqrt}(2)$

Closed loop transfer function, $T(s) = G(s)/(1+G(s)) = K/(s^2+ps+K)$

$= \omega_n^2/(s^2+ps+\omega_n^2)$
Therefore, $K=\omega_n^2=2$
$p=2\xi\omega_n=2$

(b)

E(s) =R(s)/(1+G(s))= R(s)/[1+(Km/(sTm+1)] (Answer)

Steady-state error =ess= $\lim_{s\to 0} sR(s)/(1+G(s)) = 25/(1+G(0))=25/(1+Km)=1$

Therefore, Km=24 (Answer)

TOP

Problem 2.7 **Time domain performance specifications**

A second-order servo has unity feedback and an open-loop transfer function

$$G(s) = \frac{500}{s(s+15)}$$

(a) Draw a block diagram of the closed-loop system
(b) What is the characteristic equation of the closed-loop system?
(c) Find the damping ratio, ξ and the natural frequency, ω_n.
(d) Find the time from start to the peak value of the transient response of the system to a step input.
(e) Find the settling time of the system for the step input.

Solution:

(b) TF=G/(1+G) = 500/[s(s+15)+500]
Characteristic eq. $S^2+15s+500$
(c) ω=sqrt (500), $2\xi\omega_n=15$, ξ=15/2sqrt(500)
(a) $T_p= \pi/(\omega_n \text{sqrt}(1-\xi^2))$ =0.15 sec.
(b) $T_s = 4/\xi\omega_n$ =0.53 sec.

TOP

Objective type questions:

(i) The steady state error of a control system can be minimized by

(a) increasing the gain K
(b) decreasing the gain K
(c) decreasing the natural frequency of the system.

Ans: (a)

(ii) The system function V(s)/I(s) has a pole at s = -2 and a zero at s =-1. For a sinusoidal current excitation, the voltage response will

(a) lead with respect to the current.
(b) lag with respect to the current.
(c) be in phase with the current.

(d) be zero.

Ans: (c)

(iii)The number of pure integrations in the system transfer function determine the

(a) transient performance of the system
(b) stability of the system
(c) degree of stability
(d) steady-state performance of the system.

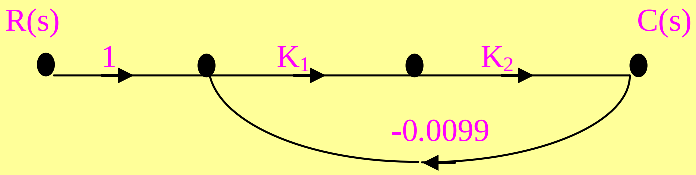
Ans: (d)

(iv) The frequency range over which the response of a system is within acceptable limits is called the system

(a) carrier frequency
(b) modulation frequency
(c) bandwidth
(d) demodulation frequency.

Ans: (c)

(v) A feedback system is shown below in signal-flow diagram form.



The sensitivity of the transfer function, T= C(s)/R(s), of the system with respect to the parameter K_1 for the nominal values of $K_1 = K_2 = 100$ is

(a) $S^T_{K1} = 0.001$
(b) $S^T_{K1} = 0.01$
(c) $S^T_{K1} = 0.1$

Ans: (b)

(vi) To reduce the contribution of the large initial error to the value of the performance index (PI) of a feedback control system, as well as to place an emphasis on errors occurring later in the response, the following PI may be used.

(a) $\int_0^T e^2 (t)\, dt$

(b) $\int_0^T |e (t)|\, dt$

(c) $\int_0^T t\,|e (t)|\, dt$

where e (t) is the error signal, and T is a finite time chosen so that the integral approaches a steady-state value.

Ans: (c)

(vii) The dynamics of a motorcycle and an average rider is represented by the open-loop transfer function

$$GH(s) = K/[s(s+20)(s^2+10s+125)]$$

The acceptable range of K for a stable system so that the vehicle can be easily controlled is

(a) $0< K < 1.069$
(b) $0< K < 10.69$
(c) $0< K < 106.9$

Ans: (b)

(viii) If ξ and ω_n are respectively the damping ratio and natural frequency of a second-order system, then for a step input

(a) the settling time of the response is equal to $4/\xi\omega_n$
(b) the time to peak of the response is equal to $4/\xi\omega_n$
(c) the percent overshoot of the response is equal to $4/\xi\omega_n$
(d) the percent overshoot of the response is equal to $\pi/(\omega_n\sqrt{(1-\xi^2)}$

Ans:(a)

(ix) It is desirable to avoid the use of the differentiator in control system design, because

(a) It is not economical
(b) Its size is big
(c) It develops noise and will saturate the amplifier
(d) None of the above.

Ans: (c)

(x) When the initial conditions are inherently zero, physically it means that
(a) The system is at rest but stores energy
(b) The system is working but does not store energy
(c) The system is at rest and no energy is stored in any of its parts
(d) The system is working with zero reference input.

Ans: (c)

(xi) The roots corresponding to the impulse response of a system, shown in Fig.4,

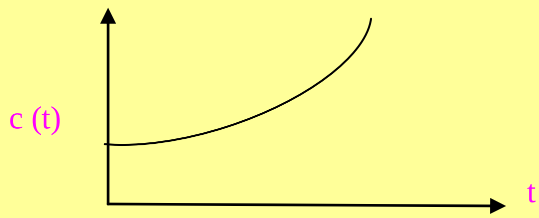


Fig.4

are

(a) $-\sigma + j\omega$, $-\sigma - j\omega$
(b) $-\sigma$
(c) σ
(d) $+ j\omega$, $- j\omega$

Ans: (c)

(xii) If the gain of the critically damped system is increased, it will

(a) become an underdamped system
(b) become an overdamped system
(c) become an oscillatory system
(e) remain a critically damped system.

Ans: (c)

(xiii) The second-order derivative of the input signal adjusts

(a) The system gain
(b) The system damping
(c) The system time constant
(d) The system time constant and suppresses the oscillations.

Ans: (d)

(xiv) If the gain in a control system is increased,

(a) The roots move away from the zeroes
(b) The roots move away from the poles
(c) The position of the roots is not affected
(d) The roots move nearer to the zeroes

Ans: (b)

(xv) The type zero system has

(a) Zero steady-state error
(b) Small steady-state error
(c) High gain constant
(d) Large steady-state error with high gain constant.

Ans: (d)

(xvi) Which one of the following statements is not true?

(a) Feedback does not reduce the sensitivity of the control system to variations in its parameters.

(b) Sensors in the feedback path may introduce noise in the system
(c) Feedback may lead to instability in the closed-loop system even if the open-loop system is stable
(d)Feedback increases the overall gain of the system.

Ans: (a)

(xvii) The transfer function of a servo system is

$$T(s) = \frac{K}{Js^2 + fs + K}$$

$J = 5 \times 10^{-2}$ Kg-cm^2 , $f = 2.5 \times 10^{-4}$ N-m per rad/sec.

The system can be made critically damped by setting gain K equal to

a. 50
b. 6.89
c. 0.5
d. 3.125×10^{-3}

Ans: (d)

(xviii) The system function V(s)/I(s) has a pole at s = -2 and a zero at s = -1. For a sinusoidal current excitation the voltage response

(a) leads the current input
(b) lags the current input
(c) is in phase with the current input
(d) is zero.

Ans: (d)

(xix) The settling time of the second-order linear system is

(a) $\tau/4$
(b) 2τ
(c) 3τ
(d) 4τ

where τ is the time constant of the system.

Ans: (b)

(xx)State *True* or *False*

(a)The time response of a second order system with ramp input tracks it with zero steady state error.
True

(b) Integral control is employed to reduce the steady state error.
False

(xxi) The open-loop transfer function of a unity feedback system representing a heat treatment oven is

$$G(s) = 20000/[(s+1)(0.5s+1)(0.005s+1)].$$

The set point (desired temperature) is 1000 degrees. The steady –state temperature is

(a) 1000
(b) 20000
(c) 1000- (1000/20001).

Ans: (c)

(xxi) The transfer function of a servo system is

$$T(s) = \frac{K}{Js^2 + fs + K}$$

J= 5 x 10^{-2} Kg-cm^2 , f = 2.5 x 10^{-4} N-m per rad/sec.

The system can be made critically damped by setting gain K equal to

a. 50
b. 6.89
c. 0.5
d. 3.125 x 10^{-3}

Ans:a

(xxii) In a feedback control system, all poles on the imaginary axis are simple (not repeated) and none of these poles are present in the input. The system is

(a) Non-asymptotically stable
(b) Asymptotically stable
(c) Unstable due to resonance
(d) Unstable

Ans:b

(xxiii) If at least some of the system poles are on the imaginary axis, the rest being in the left half of the s-plane, and if at least one of the poles on the imaginary axis is present in the form of a pole in the input, then the system is

(a) unstable
(b) unstable due to resonance
(c) non-asymptotically stable.

Ans : (b)

(xxiv) Which one of the following statements is not true?

(a) The terms ' servomechanism ' and 'position (or, velocity- or acceleration-) control system' are synonymous.
(b)A home heating system in which a thermostat is a 'controller ' is an example of servomechanism.
(c) An automatic regulating system in which the output is a variable such as temperature or pressure is called a 'process control system.'
(d) Feedback control systems are not limited to the field of engineering but can be found in various non-engineering fields such as economics and biology.

Ans: (b)

(xxv) In a d.c motor system, if Jm and J_L are the moments of inertia of the rotor and the load respectively and if N is the gear ratio then the torque to inertia ratios referred to the motor shaft and the load shaft
(a) are the same
(b) differ from each other by a factor of N
(c) differ from each other by a factor of N^2
(d) differ from each other by a factor of N^3

Ans: (b)

(xxvi) Which one of the following statements is not true?
(a) Feedback does not reduce the sensitivity of the control system to variations in its parameters.
(b) Sensors in the feedback path may introduce noise in the system
(c) Feedback may lead to instability in the closed-loop system even if the open-loop system is stable
(d)Feedback increases the overall gain of the system.

Ans: (d)

(xxvii) A speed control unity feedback system has the open –loop transfer function

$$G(s) = \frac{10}{(1+ 5 s)}$$

Its steady-state error for a unit step input is

(a) 0
(b) 1/11
(c) 1/10
(d) Infinity

Ans: (b)

(xxviii) The open-loop transfer function of a unity feedback system representing a heat treatment oven is

$G(s) = 20000/[(s+1)(0.5s+1)(0.005s+1)]$.

The set point (desired temperature) is 1000 degrees. The steady –state temperature is

(d) 1000
(e) 20000
(f) 1000- (1000/20001)
(g) 20000-(1000/20001)

Ans: (c)

TOP