Nyquist Criterion

Stability using Nyquist Criterion	Nyquist plots and the effects of additional poles and zeros on GH(s) = k/(1 + sT_1)	Nyquist criterion, Bode diagrams , and Nichol's diagram	Stability using the Nyquist Criterion(1)	Stability using the Nyquist Criterion(2)
Stability, Gain margin, Phase margin, and Type number of two systems whose Nyquist plots are given	Critical value of gain for stability using Nyquist Criterion			Objective type questions:

WEBSITE takes you to start page after you have read this chapter.
Start page has links to other chapters.

Problem: 6.1 Stability using Nyquist Criterion

(a) Fig. 1 shows a closed- loop system. Determine the critical value of K for stability by use of the Nyquist stability criterion.

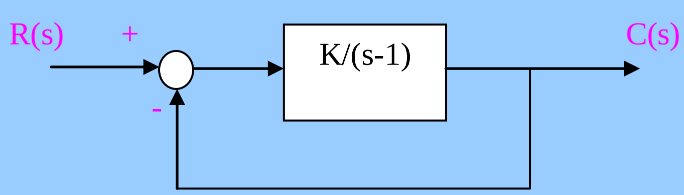


Fig.1

(b) (i) Is the system with the following open-loop transfer function and with K = 2 stable?

$$G(s)\,H(s) = K/[s(s+1)\,(2s+!)]$$

(ii) Find the critical value of the gain K for stability of the open loop system of Part (b) (ii).

Solution:

(a)

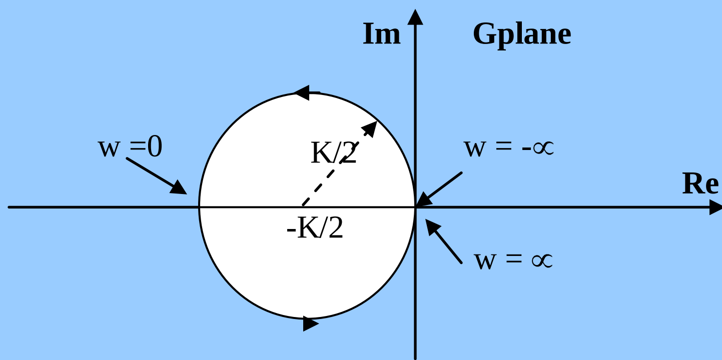
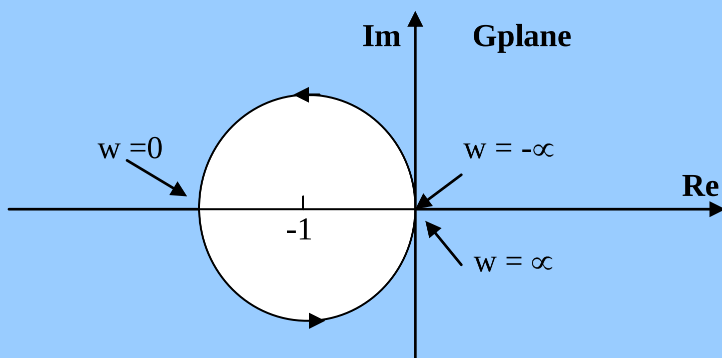


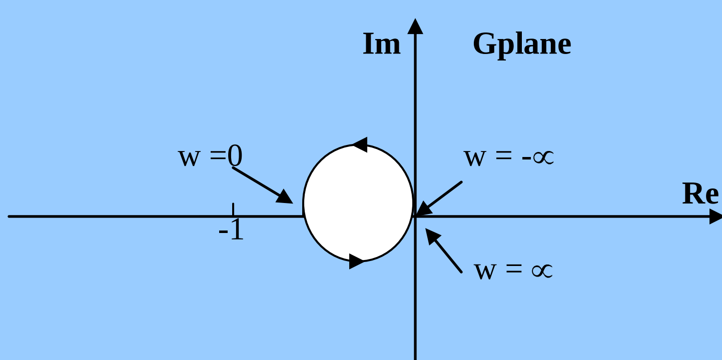
Fig.(a)



P =1
N =-1
Z=0

(STABLE)

Fig.(b)



P =1
N = 0
Z = 1

(UNSTABLE)

Fig.(c)

The polar plot of G (jw) = K/ (jw -1) is shown in Fig. (a). It is a circle with centre at –K/2 and radius K/2. P = poles in RHP=1. For the closed loop system to be stable, Z must = 0. Therefore N = Z-P = -1. There must be one counterclockwise encirclement of the -1 point for stability. Thus for stability critical K must be greater than 1. Figs (b) and (c) show the stable and unstable cases.

(b)

(i)G (jw) H (jw) = K/ [jw (jw+1) (2jw+1)] =K/ [$-3w^2$+jw ($1-2w^2$)]

Open loop transfer function has no poles in RHS of s plane. For stability, the Nyquist plot must not encircle the -1, 0 point. Find the point where the Nyquist plot crosses the negative real axis.

$1-2w^2 = 0$ or w = $\pm$ 1/sqrt (2)

Substituting w =1/sqrt (2) into G (jw) H (jw) we get

G (j/sqrt (2)) H (j/sqrt (2)) = -2K/3.

Critical K is obtained from -2K/3 =-1 or K =3/2.

System is stable when K lies between 0 and 3/2.

(ii) Hence system is unstable for K =2

TOP

Problem 6.2 : Nyquist plots and the effects of additional poles and zeros on GH(s) = k/(1 + sT_1)

Discuss the effects of additional poles and zeros of GH(s) = K/(1+sT_1) by sketching the Nyquist plots of

(i) GH(s) = K/(1+sT_1)

(ii) GH(s) = K/[s(1+sT_1)]

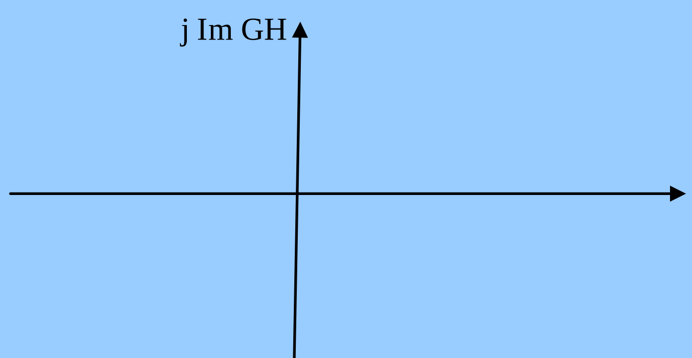
(iii) GH(s) = K/[s^2(1+sT_1)] and GH(s) = K/[s^3(1+sT_1)] on a single diagram

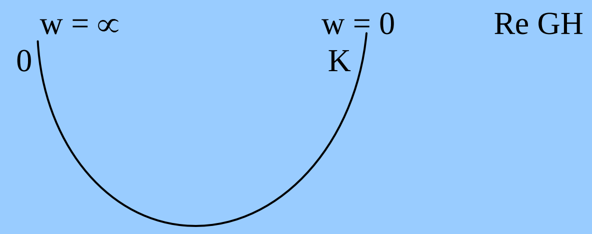
(iv) GH(s) = K/[(1+sT_1)(1+sT_2)] and GH(s) = K/[(1+sT_1)(1+sT_2)(1+sT_3)] on a single diagram

(v) GH(s) = K/[s(1+sT_1)(1+sT_2)] and GH(s) = [K(1+sT_d)]/[s(1+sT_1)(1+sT_2)] on a single diagram.

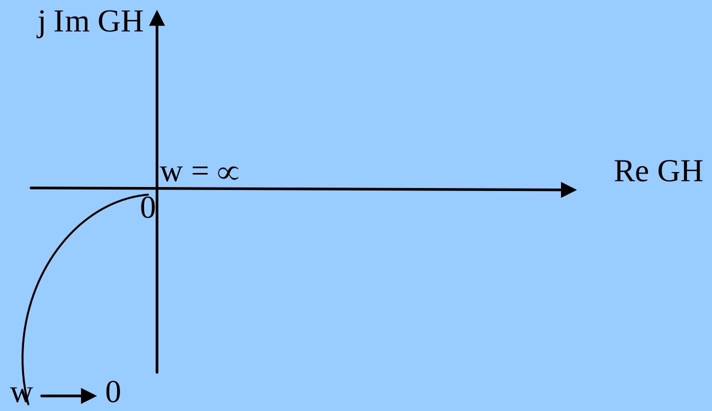
Solution:

(i)



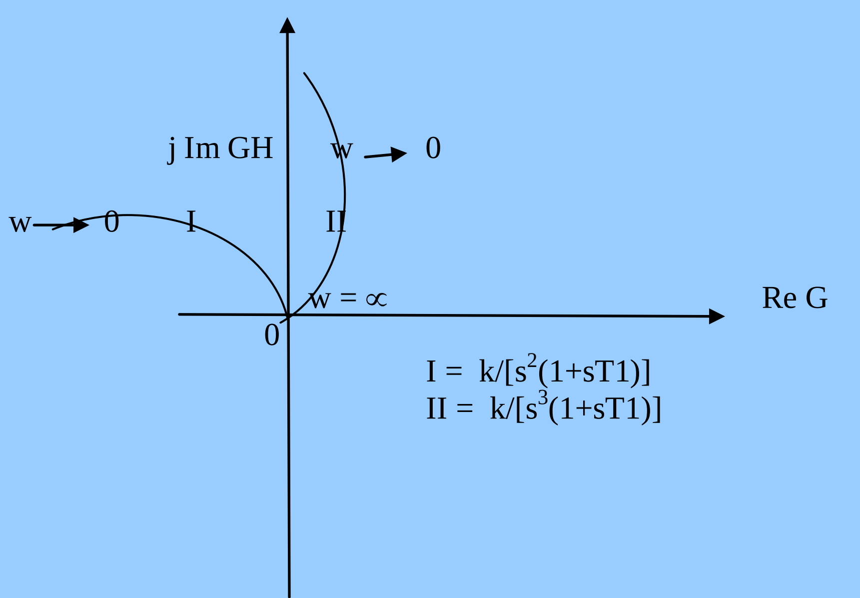


(ii)



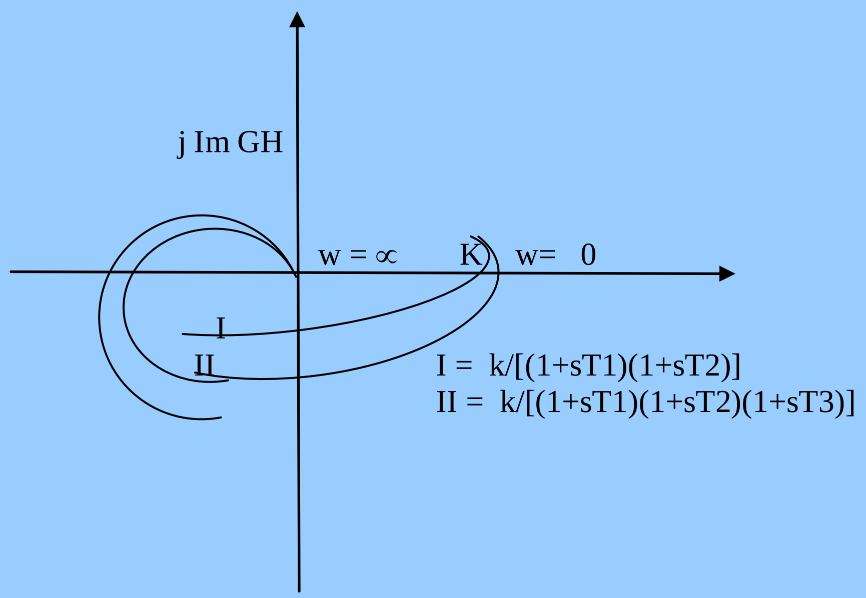
Addition of poles at s=0 will affect the stability adversely.

(iii)



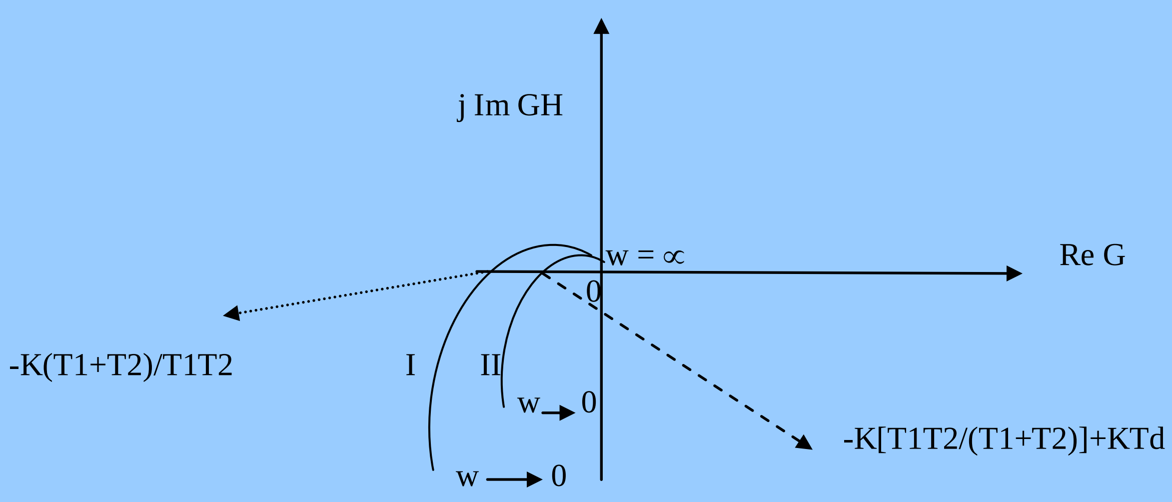
Systems with a loop transfer function of more than one pole at s=0 are likely to be unstable.

(iv)



Shows adverse effects on stability that result from addition of poles

(v)



I = K/[s(1+sT1)(1+sT2)]
I I = [K(1+sTd)]/[s(1+sT1)(1+sT2)]

Locus at w=0 in I is not affected by addition of zero. The crossover point on the real axis is moved (in II) which is closer to the origin.

TOP

Problem: 6.3 Nyquist criterion, Bode diagrams , and Nichol's diagram

(a) The open loop transfer function of unity feedback system is

$$G(s) = K/(s-1).$$

i Determine the critical value of K for stability using the Nyquist stability criterion.
ii Draw the polar plots of the system for both the stable and unstable cases.

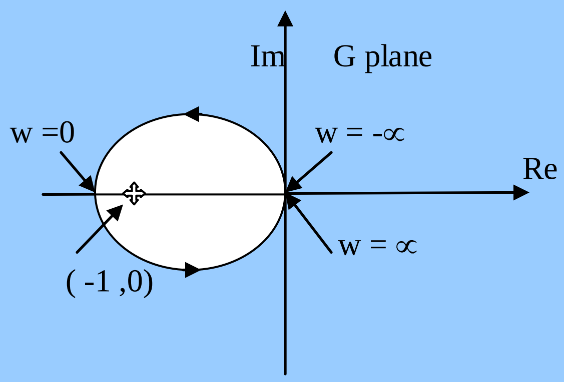
(b) For the transfer function

$$G(s) = K/(s+1)$$

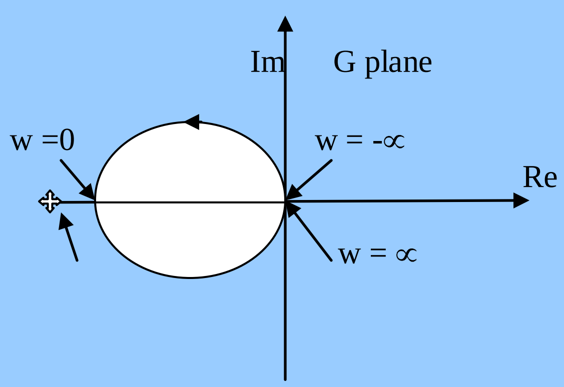
i Sketch the polar plot for $-\infty<\omega<\infty$.
ii For K = 1, sketch the Bode diagrams showing the values of magnitude, phase angle and phase margin at $\omega = 1$.
iii For K =1, sketch the Nichol's diagram showing the value of the phase margin
iv Sketch the root locus
v What is the gain margin of the system? Is the system stable?

Solution:

(a)



STABLE , K>1
P = 1
N = -1
Z =0



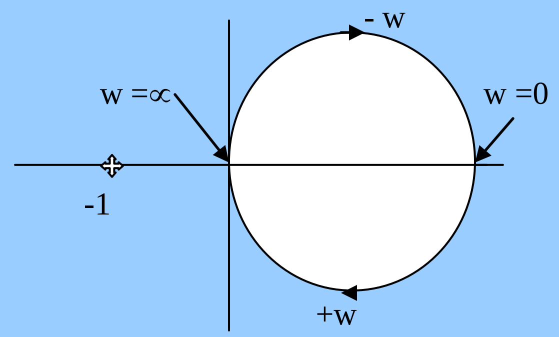
UNSTABLE, K < 1
P=1
N=0
Z=1

(-1 ,0)

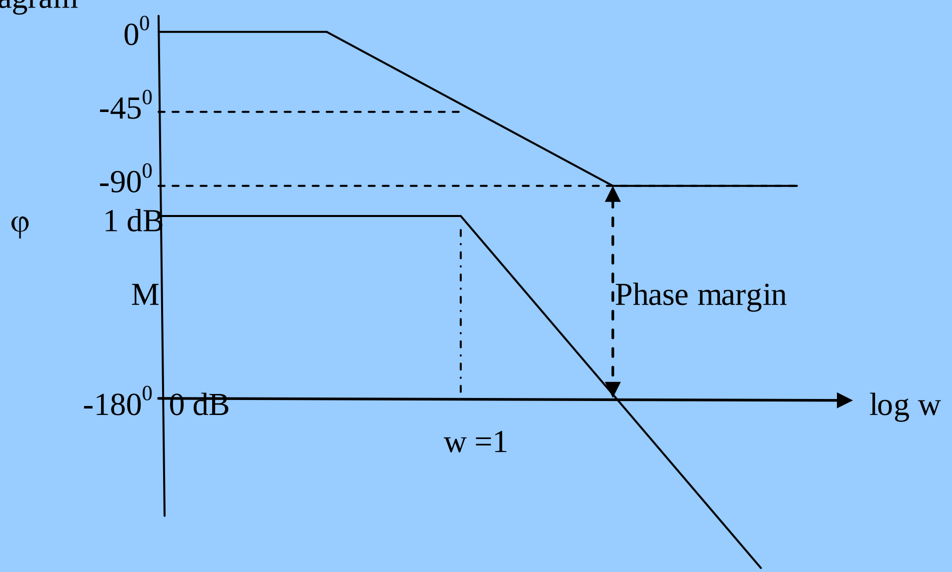
The polar plot is a circle with centre at –K/2 on the negative real axis and radius K/2.
P= no. of poles in RHP=1. For the closed loop system to be stable, Z must be equal to zero. Therefore N must be = -1 or there must be one counterclockwise encirclement of the -1, 0 point for stability. Thus, for stability, K must be greater thane 1. $K_{crit} = 1$

(b)

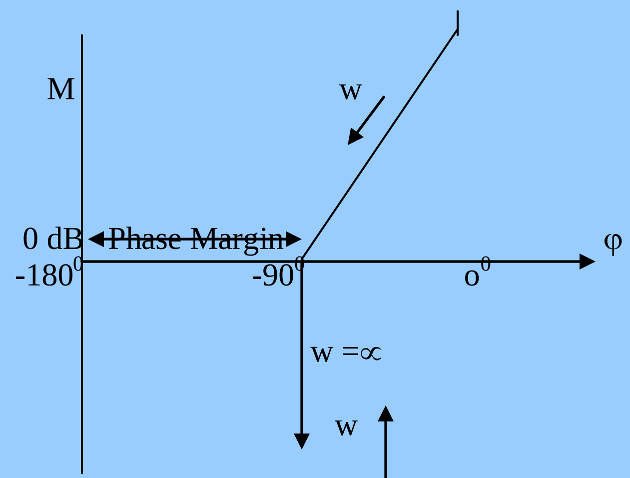
(i) Polar plot



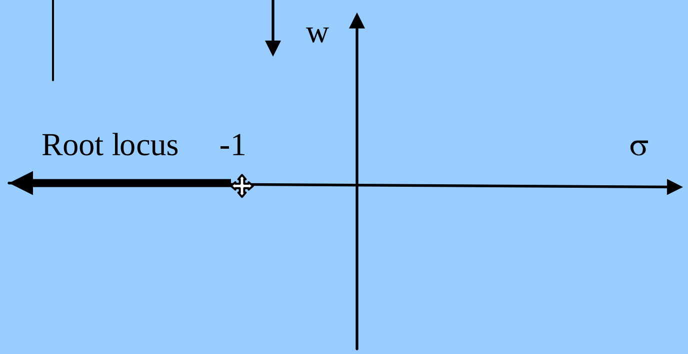
(ii) Bode diagram



(iii) Nichol’s diagram



(iv)Root locus



(v) Stable, gain margin = ∝

TOP

Problem: 6.4: Stability using the Nyquist Criterion(1)

The transfer function of the forward path of a unity feedback system is given by

$$G(s) = \frac{10(s+5)}{s^2}$$

Sketch the polar plot of the frequency response at ω = 0.1, 1, 2, 5, 10, 20, 30, 50, 100, and use the Nyquist criterion to investigate the stability of the system

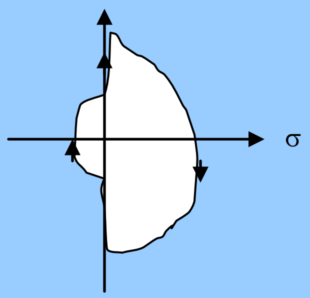
Solution:

The following table shows the frequency response:

ω	0.1	1	2	5	10	20	30	50	100
Gain	5001	50.99	13.46	2.828	1.12	0.515	0.338	0.201	0.1
φ phase	-178.9	-168.3	-158.2	-135	-116.6	-104	-99.5	-95.75	52.9

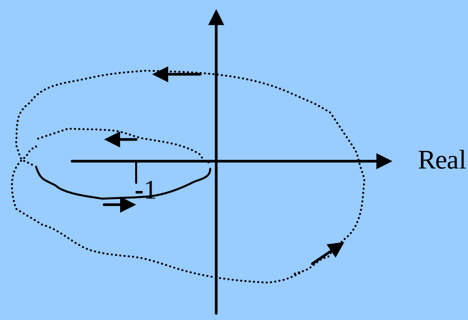
(deg.)									

The figure on the left shows the Nyquist contour and that on the right shows the Nyquist plot.



s-plane

P = 2



GH plane
N = -2

From the plot, N = -2, since P= 2, the system is stable.

TOP

Problem: 6.5 Stability using the Nyquist Criterion(2)

Draw the Nyquist contour plot of a single–loop unity feedback control system when

$$G(s) = \frac{K}{s^2(s+1)}$$

Determine the stability of the system from the Nyquist plot.

Solution:

$GH(s) = K/s^2(s+1)$
$GH (jw) = - K /w^2 (jw+1)$
$Mag.GH (jw) = K/sqrt (w^4+w^6)$
$Angle\ GH (jw) = -\pi-\tan^{-1}w$

Angle is always –180 deg. or greater. Therefore, locus of GH (jw) is above the real axis for all w.
As w approaches 0+,
$Lim\ GH (jw) = \lim_{w \to 0+} \left| K/w^2 \right| \angle -180$ deg.

As w approaches infinity,

$$\lim GH(jw) = \lim_{w \to \infty} |K/w^2| \angle -3\pi/2$$

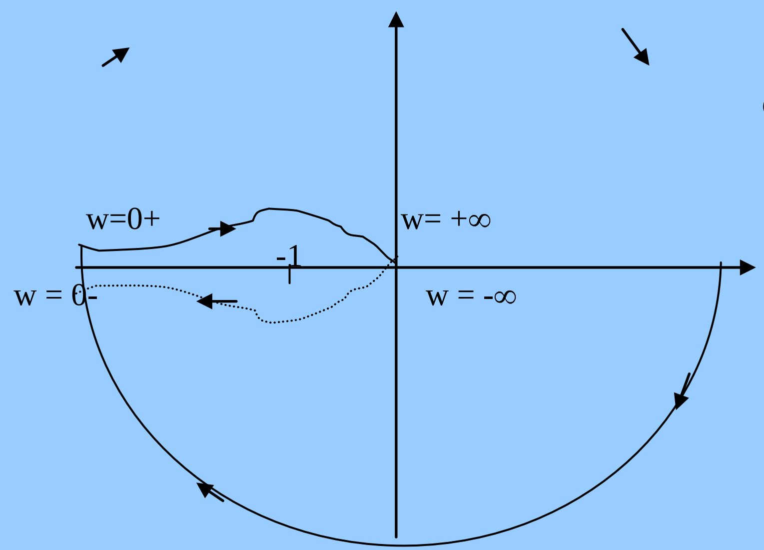
At the small semi-circular detour at the origin, s = ε exp (ϕ) and

$$\lim GH(s) = \lim_{\varepsilon \to 0+} |K/\varepsilon^2| \angle -2\phi$$

where $-\pi/2 \leq \phi \leq \pi/2$

The complete contour is shown in Fig. below.

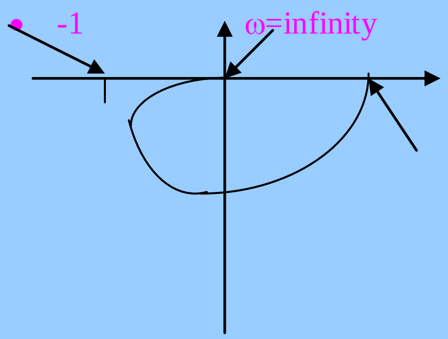
Because the contour encircles the –1-point twice, there are two roots of the close loop system in the RH plane and the system irrespective of the gain K is **unstable.**

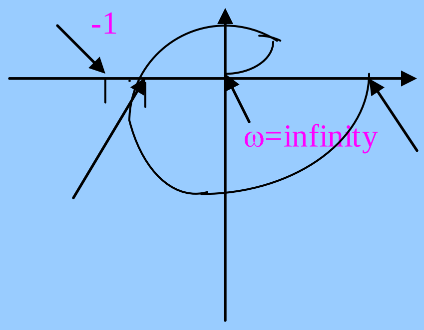


TOP

Problem 6.6: Stability, Gain margin, Phase margin, and Type number of two systems whose Nyquist plots are given

The polar Nyquist plots shown in Figs (2) and (3) for two systems are sketches of the map of the positive imaginary axis of the s-plane. None of the G(s) functions has poles in the RHP





+ω ω=0 +ω ω=0

-0.7

Fig.2 Fig.3

(i) Complete each plot, i.e.; add the map of the negative imaginary axis.
(ii) Is each system stable?
(iii) Calculate and indicate the phase margin of each system on the plot.
(iv) Calculate and indicate the gain margin of each system on the plot.
(v) What is the Type number of each system?

Solution:

(a)

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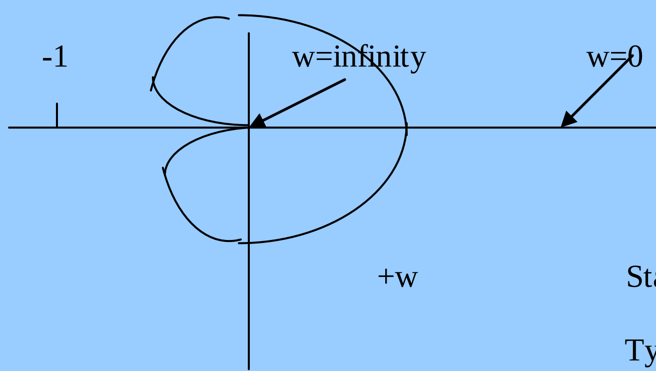


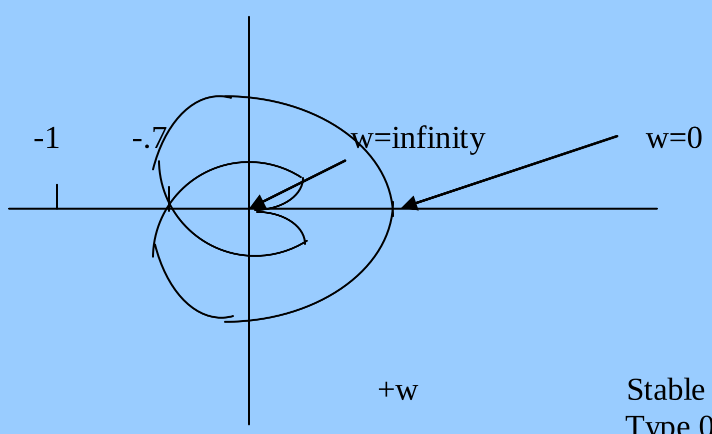
P.M = 30 deg
G.M= infinity

Stable

Type 0

-w

•



A

P.M = 20 deg

Stable
Type 0

G.M= 1/OA=1/.7=1.43

TOP

Problem 6.7 : Critical value of gain for stability using Nyquist Criterion

The open-loop transfer function of a control system is

$$G(s)H(s) = \frac{K}{s(s+1)(2s+1)}$$

(a) Determine if the system is stable with K=2.

(b) Use the Nyquist stability criterion and determine the critical value of the gain K for stability

Solution:

OLTF is

$G (jw) H (jw) = k/[jw (jw+1)(2jw+1)] =K/[-3w^2 +jw (1-2w^2)]$

This OLTF has no poles in the RHP. For stability, -1+j0 should not be encircled by the Nyquist plot. Find the point at which the Nyquist plot crosses the real axis. For this, the imaginary part of G (jw) H (jw) =0 or, $1-2w^2=0$ or w=(+/-) 1/sqrt (2).
Therefore, G (j/sqrt2) H (j/sqrt2)=-2K/3. Critical value of K is obtained from-2K/3=-1.
K_{crit}= 1.5
System is stable for 0< K< 1.5
Unstable for K=2

TOP

Objective type questions:

Choose the correct answers from those given below:

(i) The negative real axis of the Nyquist diagram transforms into
- (a) a negative 180^0 phase line of Bode diagram for all frequencies
- (b) the unity or 0 dB line of Bode amplitude plot
- (c) the unity or 0 dB line of Bode amplitude plot for low frequencies
- (d) a negative 180^0 phase line of Bode diagram for high frequencies

Ans: (d)

(ii) Fig.6 shows the Nyquist plot of a control system.

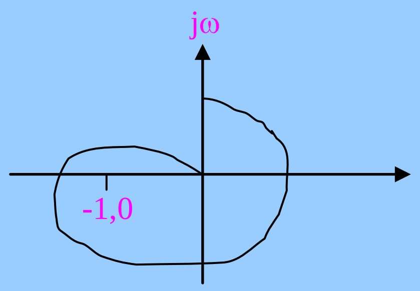


Fig.6

What will be the type of the system?

(a) Type3

(b) Type 2

(c) Type 1

(d) Type 0

Ans:(c)

(iii) The unit circle of the Nyquist plot transforms into the 0 dB line of the Bode amplitude plot for

a. low frequencies
b. high frequencies
c. all frequencies
d. none of the above.

Ans: (b)

(iv) In applying Nyquist criterion for stability, which one of the following possibilities is not true?

(a) The system is <u>unstable</u> if the number of <u>counterclockwise</u> encirclements of the –1+j0 point is the same as the number of poles of G(s)H(s) in the right-half s- plane.

(b) The system is <u>stable</u> if there are no encirclement of the –1+j0 point and if there are no poles of G(s)H(s) in the right-half s-plane

(c) The system is stable if the number of counterclockwise encirclements of the –1+j0 point is the same as the number of poles of G(s)H(s) in the right-half s- plane.

(d) The system is unstable if there is/are a clockwise encirclement /encirclements of the –1+j0 point.

Ans: (a)

TOP