Problem 1 Matrices and differential equations

(a) Find matrices X and Y such that AX = B and YA = B where

\[
A = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix}, \quad B = \begin{pmatrix} 3 & 2 \\ -6 & -5 \end{pmatrix}
\]

(b) Determine the solution of the following set of first-order coupled differential equations:

\[
\begin{align*}
\frac{dx_1}{dt} &= x_2 \\
\frac{dx_2}{dt} &= -6x_1 - 5x_2
\end{align*}
\]

when the initial conditions are \(x_1(0) = 1\) and \(x_2(0) = 0\).

Solution

(a)

\[
X = \begin{pmatrix} 0 & -1 \\ 3 & -4 \end{pmatrix}
\]
Problem 2 Solution of second-order differential equation

(a) Determine the solution of the following differential equation when all the initial conditions \( y(0) \) and \( \frac{dy}{dt} \bigg|_{0} \) are zero and \( f(t) = 4t \):

\[
d\frac{2}{dt}y + 3\frac{dy}{dt} + 2y = f(t)
\]

(b) Determine the inverse transformation of the following transformed equation:

\[
Y(s) = \frac{75}{(s^2 + 4s + 13)(s + 6)}
\]

Solution

(a)

Answer: \( y = 4e^{-t} - e^{-2t} + 2t - 3 \)

(b)

Answer: \( y = e^{-2t} (4 \sin{3t} - 3\cos{3t}) + 3 e^{-6t} \)

Or: \( y = 3e^{-2t} \sin{(3t - 36.9^\circ)} + 3 e^{-6t} \)
Problem 3  Partial fraction expansions

Find the partial fraction expansions of the following algebraic fractions:

(a) \( F(x) = \frac{2(x^2+3x+1)}{x(x^2+3x+2)} \)

(b) \( F(x) = \frac{2x+3}{(x+1)(x^2+4x+5)} \)

(c) \( F(x) = \frac{1}{x^2(x+1)} \)

Solution

(a) Answer: \( \frac{1}{x} - \frac{2}{x+1} - \frac{1}{x+2} \)

(b) Answer: \[ \frac{0.25+j.75}{x+2+j1} + \frac{-0.25-j.75}{x+2-j1} + 0.5/(x+1) \]

(c) Answer: \( \frac{1}{x^2} - \frac{1}{x} + \frac{1}{x+1} \)

TOP

Problem 4  Determinant, inverse and eigenvalues of a matrix

(a) Find the determinant and inverse of the following matrix.

\[
A = \begin{pmatrix}
-2 & 3 & 1 \\
4 & 6 & -2 \\
8 & 4 & 1 \\
\end{pmatrix}
\]

(b) Find the eigenvalues of the following matrix.

\[
B = \begin{pmatrix}
-2 & -5 & -5 \\
1 & -1 & 0 \\
0 & 1 & 0 \\
\end{pmatrix}
\]

(c) Find \( \exp (-Ct) \) for the following matrix.

\[
C = \begin{pmatrix}
0 & 1 \\
-2 & -3 \\
\end{pmatrix}
\]
Solution

(a) Determinant of A = -120

Inverse of A = \[
\begin{pmatrix}
14 & -1 & -12 \\
-20 & 10 & 0 \\
-32 & -32 & -24 \\
\end{pmatrix}
\] - \((1/120)\)

(b) Eigenvalues of B = -1, -1+j2, -1-j2

(c) \(\exp(Ct) = L^{-1} [(sI-C)^{-1}]\)

\(sI - C = \begin{pmatrix}
s & -1 \\
2 & s+3 \\
\end{pmatrix}\)

\((sI-C)^{-1} = (1/ (s^2+3s+2)). \begin{pmatrix}
s+3 & 1 \\
-2 & s \\
\end{pmatrix}\)

\(= \begin{pmatrix}
[2/(s+1)] - [2/(s+2)] & [1/(s+1)] - [1/(s+2)] \\
[2/(s+2)] - [2/(s+1)] & [2/(s+2)] - [1/(s+1)] \\
\end{pmatrix}\)

\(\exp(Ct) = L^{-1} [(sI-C)^{-1}] = \)

\(= \begin{pmatrix}
2e^{-t} - e^{-2t} & e^{-t} - e^{-2t} \\
2e^{-2t} - 2e^{-t} & 2e^{-2t} - e^{-t} \\
\end{pmatrix}\)
**Problem 5  Solution of first-order coupled differential equations**

(a) Describe a procedure to solve a set of first order coupled differential equations

(b) Solve the following system of linear differential equations with constant coefficients:

\[
\begin{align*}
3 \frac{dx_1}{dt} + 2x_1 + dx_2 &= 1 \quad t \geq 0 \\
\frac{dx_1}{dt} - 3x_2 + 4 \frac{dx_2}{dt} &= 0
\end{align*}
\]

The initial conditions are:

\[
\begin{cases}
x_1 = 0 \quad \text{at} \quad t = 0 \\
x_2 = 0
\end{cases}
\]

**Solution**

(b) Let \( L\)-Transform of \( X_1 = Y_1 \)

Let \( L\)-Transform of \( X_2 = Y_2 \)

Transform the differential equations to

\[
\begin{align*}
(3s+2) Y_1 + sY_2 &= 1 \\
sY_1 + (4s+3) Y_2 &= 0
\end{align*}
\]

Solve these two equations to obtain

\[
Y_1 = \frac{(4s+3)}{(s+1)(11s+6)} = L (X_1)
\]

Partial fractions and Laplace inverse gives

\[
X_1 = 0.5 - 0.2e^{-t} - 0.3e^{-6t/11}
\]

Similarly,

\[
X_2 = 0.5(e^{-t} - e^{-6t/11})
\]

TOP
**Problem 6  Transfer function of a multiloop feedback control system**

(a) Fig. 1 shows a multiple loop feedback control system.

![Diagram of a multiple loop feedback control system](image)

Fig. 1

Determine the transfer function of the system, $C(s)/R(s)$, when $G_1 = G_2 = G_3 = G_4 = 1$, and $H_1 = H_2 = H_3 = 1/s$.

(b) Fig. 2 shows a two-path signal–flow graph of a multi-legged robot.

![Diagram of a two-path signal–flow graph](image)

Fig. 2

Use Mason’s loop rule to determine the transfer function, $C(s)/R(s)$. 
Solution
(a) Using block diagram reduction of the system, we get

Transfer function = \( \frac{C(s)}{R(s)} = \frac{s}{s+1} \) Ans.
(b) path1: \( P1 = G1G2G3G4 \)

Path2: \( P2 = G5G6G7G8 \)

Self-loops: \( L1 = G2H2, \ L2 = G3H3, \ L3 = G6H6, \ L4 = G7H7 \)

Determinant, \( \Delta = 1 - (L1+L2+L3+L4)-(L1L3+L1L4+L2L3+L2L4) \)

Co-factors: \( \Delta 1 = 1 - (L3+L4), \quad \Delta 2 = 1 - (L1+L2) \),

\[
C(s)/R(s) = \left( P1\Delta 1 + P2\Delta 2 \right) / \Delta \\
= \left[ G1G2G3G4(1-L1-L4)+G5G6G7G8(1-L1-L2) \right] / (1-L1-L2-L3-L4+L1L3+L1L4+L2L3+L2L4)
\]

**Problem 7** Feedback model of a system

(a) The student–teacher learning process is inherently a feedback process intended to reduce the system error to a minimum.

The desired input is the knowledge being studied and the student may be considered the process. With the aid of Fig. 3, construct a feedback model of the learning process and identify each block of the system.

![Feedback diagram](image-url)
(b) Consider the pendulum of Fig. 4. The torque on the mass is

\[ T = MgL \sin \theta \]

where \( g \) is the gravity constant. The equilibrium condition for the mass is \( \theta_0 = 0^\circ \). Determine, from first principles, the linear approximation of the nonlinear relation between \( T \) and \( \theta \). State the range of \( \theta \) for which the approximation is reasonably accurate.

**Solution**

(a)

\[ T = MgL \sin \theta \]

(b) \[ T = MgL \frac{\delta \sin \theta}{\delta \theta} |_{\theta = \theta_0} (\theta - \theta_0) \]

\[ = MgL (\cos \theta_0) (\theta - \theta_0) \]

\[ = MgL \theta \]
This approximation is reasonably accurate for $-\pi/4 \leq \theta \leq \pi/4$.

**TOP**

**Problem 8  Difference equation of a system**

(a) Fig. 5 shows a mechanical system, where

- $F(t)$ = Force applied to the system
- $M$ = Mass
- $B$ = Coefficient of friction
- $K$ = Spring constant
- $y$ = Displacement of the mass.

Write the state equations of the system, choosing displacement and velocity of the mass as state variables, $x_1$ and $x_2$, respectively.

![Mechanical System Diagram](image)

(b) Write the difference equation obeyed by a sum of money in a bank account at a yearly interest rate of $R$ percent, compounded every $T$ years. Denote the amount immediately following the $k$th interest period as $A(kT)$. Find the differential equation which results as $T \rightarrow 0$. 

---
Solution

(a) Force absorbed by mass, $f_M = M \frac{d^2 y}{dt^2}$

Force absorbed by friction, $f_B = B \frac{dy}{dt}$

Force absorbed by spring, $f_s = Ky$

$f_M + f_B + f_s = F$

Let $x_1 = y$, $x_2 = \frac{dy}{dt}$

The state equations are:

$$\frac{dx_1}{dt} = x_2$$

$$\frac{dx_2}{dt} = -\frac{Kx_1}{M} - \frac{Bx_2}{M} + \frac{F(t)}{M}$$

(b) The principal obeys the difference equation

$$A(kT + T) = A(kT) \left[1 + \frac{RT}{100}\right]$$

Re-writing, $[A(kT + T) - A(kT)]/T = \frac{RA(kT)}{100}$.

Taking the limit as $T \to 0$ and letting $kT \to t$ gives the differential equation

$$\frac{dA}{dt} = \frac{RA(t)}{100}.$$
**Problem 9  Controller for a multiloop unity feedback control system**

Fig. 6 shows a closed-loop multivariable control system.

![Diagram](image)

The transfer function for the plant is

\[
G(s) = \begin{bmatrix} \frac{5}{1+2s} & \frac{50}{1+2s} \\ \frac{1}{1+s} & \frac{-10}{1+s} \end{bmatrix}
\]

The desired closed-loop transfer function is

\[
T(s) = \begin{bmatrix} \frac{1}{1+0.5s} & 0 \\ 0 & \frac{1}{1+0.2s} \end{bmatrix}
\]

Determine the required controller \( G_c(s) \).

**Solution**

It can be easily shown that \( G_c(s) = G^{-1} (T^{-1}-1)^{-1} \)

\[
(T^{-1}-1) = \begin{bmatrix} 0.5s & 0 \\ 0 & 0.2s \end{bmatrix}
\]

\[
(T^{-1}-1)^{-1} = \begin{bmatrix} 2/s & 0 \\ 0 & 5/s \end{bmatrix}
\]
\[
G^{-1} = \left(\frac{1}{\Delta}\right) \begin{pmatrix}
-10/(1+2s) & -50/(1+2s) \\
-1/(1+s) & 5/(1+s)
\end{pmatrix}
\]

where \(\Delta = -100/(1+2s)(1+s)\)

Therefore, \(G_c = \begin{pmatrix}
0.2(1+2s) & 2.5(1+s) \\
0.02(1+2s0 & -0.25(1+s)
\end{pmatrix}\)

\textbf{TOP}

\textbf{Problem 10} Transfer function of a two–mass mechanical system

(a) Show that a system, which gives a response \(x\) corresponding to an input \(u\), and governed by the equation

\[u = \frac{d^2x}{dt^2} + a.\frac{dx}{dt} + b.x\]

is linear. Here \(a\) and \(b\) are constants,

(b) Fig.7 shows a two-mass mechanical system.
Friction $f_2$

Spring constant $K$

Velocity $v_2(t)$

Friction $f_1$

Velocity $v_1(t)$

Force $r(t)$

Fig. 7
i Determine the transfer function, \( V_1(s)/R(s) \), if the velocity of mass \( M_1 \) is the output variable and the applied force on \( M_1 \) is the input variable.

ii What would be the transfer function when the position \( x_1(t) \) of mass \( M_1 \) is the output variable?

**Solution**

(a) \( x = \frac{d^2}{dt^2} x + a \frac{dx}{dt} + bx = T.u \), where \( T \) is the system operator.

Multiply both sides of the given equation by a scalar quantity \( k \). We have

\[
k.u = k \frac{d^2}{dt^2} x + k.a \frac{dx}{dt} + k.b x = \frac{d^2}{dt^2} (kx) + a \frac{dx}{dt} + b (kx)
\]

Thus, if \( x \) is the response of the system due to an input \( u \), then for an input \( ku \), the response will be \( kx \). Also, let \( x_1 \) be the response to \( u_1 \), and \( x_2 \) be the response to input \( u_2 \), then

\[
\begin{align*}
u_1 &= \frac{d^2}{dt^2} x_1 + a \frac{dx_1}{dt} + b x_1 \\
u_2 &= \frac{d^2}{dt^2} x_2 + a \frac{dx_2}{dt} + b x_2
\end{align*}
\]

Adding the two equations

\[
u_1 + u_2 = \frac{d^2}{dt^2} (x_1 + x_2) + a \frac{dx_1 + dx_2}{dt} + b(x_1 + x_2)
\]

Thus when, \( x_1 = T.u_1 \) and \( x_2 = T.u_2 \),

\[(x_1 + x_2) = T. (u_1 + u_2) = Tu_1 + Tu_2
\]

Therefore, the system operator is linear.

(c) (i) The simultaneous equations, assuming the initial conditions are zero, are

\[
\begin{align*}
M_1 s V_1(s) + (f_1 + f_2) V_1(s) - f_1 V_2(s) &= R(s) \\
M_2 s V_2(s) + f_1 (V_2(s) - V_1(s)) + KV_2(s)/s &= 0
\end{align*}
\]

Solve the above two equations for \( V_1(s) \) [by eliminating \( V_2(s) \)] and obtain the transfer function
\[ G(s) = \frac{V_1(s)}{R(s)} = \frac{(M_2s^2 + f_1s + K)}{(M_1s + f_1 + f_2)(M_2s^2 + f_1s + K)} \]

(ii) \[ \frac{X_1(s)}{R(s)} = \frac{V_1(s)}{sR(s)} = \frac{G(s)}{s} \]

**Problem 11  Signal-flow graph for a water level controller**

(a) Fig. 8 shows a water level controller in which the input valve closes completely when the level of water in the tank reaches a height \( h_r \).

![Signal-flow graph for a water level controller](image)

The water level in the tank at any time is \( h \) and the valve gate-opening \( x \) is caused by the arrangement of the float \( F \) and lever \( L \). The cross-sectional area of the tank is \( A \). The valve discharge \( q_i \) is proportional to the gate-opening \( x \) with proportionality factor \( k \), and the output
flow \( q_o \) is proportional to the head \( h \) with proportionality factor \( B \). Assume that the maximum input \( q_{i, \text{max}} \) corresponding to the full gate opening never exceeds the output \( q_o \). Develop a signal flow graph between the input \( q_i \) and the output \( h_i \). Assume that the Laplace transforms of \( q_i \), \( x \), \( h_r \), and \( h_i \) are \( Q_i \), \( X \), \( H_r \), and \( H_i \) respectively.

(b) Determine the transfer function, \( C/R \), for the control system shown in Fig. 9.

\[ C/R = \frac{G_1 G_2 G_3}{1 + G_1 G_2 G_3 + G_3 H_1} \]

---

**Solution**

(a) (i) \( q_i = Adh_i/dt + Bh_i \)

\( x = (h_r - h_i) \) (a/b)

\( q_i = kx \)

Taking the Laplace transforms, we have after rearranging

\( X = (H_r - H_i) \) (a/b)

\( Q_i = kX \)

\( H_i = Qi/ (As+B) \)

---

Fig. 9
Problem 12  To determine the magnitude and phase angle of $G(j\omega)$

(a) Describe a subroutine and draw a flow-chart for obtaining the magnitude and angle of $G(j\omega)$ when the transfer function is

$$G(s) = \frac{10}{s \ (1+0.25s) \ [(s^2+10s+64)/64]}.$$ 

(b) Determine the magnitude and angle of $G(j\omega)$ for the transfer function $G(s)$ of Part (a).

Solution

(b) $G(j\omega) = \frac{10}{j\omega \ (1+.25j\omega) \ (64-w^2+j10\omega)/64}$

Let $G(j\omega) = 10G_1(j\omega) \ G_2(j\omega) \ G_3(j\omega)$

Mag.$G_1(j\omega) = 1/w$

Mag.$G_2(j\omega) = 1/\sqrt{(1+(.25\omega)^2)}$

Mag.$G_3(j\omega) = 64/\sqrt{((64-w^2)+(10\omega)^2)}$

Angle $G_1(\omega) = -45^\circ = -1.57 \text{ rad}$.

Angle $G_2(\omega) = -\tan^{-1}0.25\omega \text{ rad}$.

Angle $G_3(\omega) = -\tan^{-1} \frac{10\omega}{(64-w^2)}$

TOP

Problem 13  Solution of a second-order differential equation

(a) Find the 2 x 2 matrices $X$ and $Y$ such that $AX = B$ and $YA = B$, where
\[ A = \begin{pmatrix} 2 & -1 \\ -3 & 2 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 2 \\ -6 & -5 \end{pmatrix} \]

(b) Find the eigenvalues of the matrix:

\[ A = \begin{pmatrix} 2 & 1 & 1 \\ 2 & 3 & 4 \\ -1 & -1 & -2 \end{pmatrix} \]

(c) Using the state variables: \( x_1 = y, x_2 = \frac{dy}{dt} \), determine the solution of the following differential equation:

\[ \frac{d^2 y}{dt^2} + 5 \frac{dy}{dt} + 6y = 0. \]

The initial conditions are \( y(0) = 1, \frac{dy}{dt}(0) = 0. \)

**Solution**

(a) \( A^{-1} = \begin{pmatrix} 3 & 2 \\ 2 & 1 \end{pmatrix} \)

\[ X = A^{-1}B = \begin{pmatrix} 0 & -1 \\ -3 & -4 \end{pmatrix} \]

\[ Y = B A^{-1} = \begin{pmatrix} 12 & 7 \\ -27 & -16 \end{pmatrix} \]

(b)

Eigenvalues are given by the characteristic equation

\[ \begin{vmatrix} 2-\lambda & 1 & 1 \\ 2 & 3-\lambda & 4 \\ -1 & -1 & -2-\lambda \end{vmatrix} = 0 \]

Thus, \( \lambda = 1, -1, 3 \)

(c) The state space representation is
\[ \frac{dx_1}{dt} = x_2 \]
\[ \frac{dx_2}{dt} = -6x_1 - 5x_2 \]

Laplace transform is

\[ SX_1(s) - x_1(0) = X_2(s) \]
\[ SX_2(s) - x_2(0) = -6X_1(s) - 5X_2(s) \]

Matrix \( A = \begin{bmatrix} 0 & 1 \\ -6 & -5 \end{bmatrix} \)

\[ \phi(s) = (sI - A)^{-1} = \begin{bmatrix} \frac{3}{s+2} & -\frac{2}{s+3} & \frac{1}{s+2} & -\frac{1}{s+3} \\ -\frac{6}{s+2} & +\frac{6}{s+3} & -\frac{2}{s+2} & +\frac{3}{s+3} \end{bmatrix} \]

Find \( \phi(t) \). Solution is: \( x(t) = \phi(t). x(0) \).

Therefore, \( y = 3\exp(-2t) 2\exp(-3t) \)

\[ \frac{dy}{dt} = -6(\exp(-2t) - \exp(-3t)) \]

**Problem 14**  To determine the roots of a polynomial

(a) Describe a subroutine and draw a flow-chart to determine the polynomial whose roots are located at \( s = -a, s = -b, \) and \( s = -c \) respectively.

(b) Determine the roots of the following polynomial:

\[ 4s^3 - 20s^2 + 17s + 14 = 0 \]

(c) Find the inverse Laplace transform of the following function:

\[ C(s) = \frac{s^2 + 2s + 3}{(s + 1)^3} \]
Solution

(b) 4\(s^3\)-20\(s^2\)+17\(s\)+14 = 0

The exact roots are -0.5, 2, 3.5

(c) Partial fraction expansion is \(F(s) = \frac{2}{(s+1)^3} + \frac{1}{(s+1)}\)

\(f(t) = (t^2+1) \exp(-t), t \geq 0\)

TOP

**Problem 15**: Transfer function using Mason’s Rule

(a) The block diagram of a control system is shown in Fig. 10. Using block-diagram reduction, find the transfer function, \(C(s)/R(s)\).
(b) Find the transfer function, \(C(s)/R(s)\) using Mason’s Rule also.

**Solution**

(a) Transfer function after block diagram reduction

\[
C(s)/R(s) = \frac{G_1 G_2 G_3 G_4}{1 - G_3 G_4 H_1 + G_2 G_3 H_2 + G_1 G_2 G_3 G_4 H_3}
\]

(b) Transfer function = \(P_1 \Delta_1 / (1-L_1-L_2-L_3)\)

\[P_1 = G_1 G_2 G_3 G_4\]

\[L_1 = -G_2 G_3 H_2\]

\[L_2 = -G_3 G_4 H_1\]

\[L_3 = -G_1 G_2 G_3 G_4 H_3\]

Path \(P_1\) touches all the loops. Hence \(\Delta_1 = 1\).

Transfer function is given in Part (a)

**TOP**

**Problem 16** Roots of a third-order polynomial

(a) Describe a method of determining the roots of a third-order polynomial.

(b) Determine the roots of a polynomial

\[s^3 + 4s^2 + 6s + 4 = 0\]

(c) Obtain the partial fraction expansion of

\[
\frac{2(s^2 + 3s + 1)}{s(s+1)(s+2)}
\]

**Solution**

(b):

\[F(s) = s^3 + 4s^2 + 6s + 4\]

Table of synthetic division is:

\[
\begin{array}{cccc}
1 & 4 & 6 & 4 \\
\end{array}
\]

\(-1 = \text{trial root} \)
1 -3 -3
1 3 3

1 = remainder

for a trial root of \( s = -1 \). In this Table, multiply the trial root and successively add in each column.

With a remainder of 1, try

\[ s = -2 \] which results in

\[
\begin{array}{ccc}
1 & 4 & 6 \\
-2 & -2 & -4 \\
\end{array}
\]

\[
\begin{array}{ccc}
1 & 2 & 2 \\
0 & & \end{array}
\]

Because the remainder is zero, one root is \( s = -2 \).

Let \((s+2) (s^2 +as + b) = 0\). Comparing coefficients, \( a = 2, b = 2 \). Therefore, the remaining roots are: \( s = -1+j \) and \( -1-j \).

\[
(F(s) = (1/s) + (2/(s+1)) - (1/(s+2))
\]

**TOP**

**Problem 16**

**OBJECTIVE TYPE QUESTIONS:**

Choose the correct answers from those given below:
by writing (a), (b), or (c):

(i) The partial fraction expansion of \( 2(x^2 + 3x +1) \) is

\[
\frac{A_1}{x} + \frac{A_2}{(x+1)} + \frac{A_3}{(x + 2)}
\]

where

(a) \( A_1 = 1, A_2 = -2, A_3 = -1 \)
(b) \( A_1 = 1, A_2 = 2, A_3 = 1 \)
(c) \( A_1 = 1, A_2 = -1, A_3 = -2 \)

Ans: (a)

(ii) The eigenvalues of the matrix \[
\begin{pmatrix}
0 & 1 \\
\end{pmatrix}
\]
are:

(a) –1 and –2
(b) –2 and –3
(c) –3 and –4

Ans: (b)

(iii) Which one of the following properties of the root locus is not true?
(a) A branch of the root locus starts from each open-loop pole and terminates at each open-loop zero at infinity
(b) The root locus is symmetrical about the real axis of the s-plane.
(c) If the number of finite zeros $m$ is less than the number of finite poles $n$, then $n+m$ branches of the root locus must end at zeros at infinity.

Ans: (c)

(iv) The transfer function of a system is
$$\frac{10(1+0.2s)}{(1+0.5s)}$$
The corner frequencies will be
(a) $0^0$ and $-90^0$
(b) $180^0$ and $180^0$
(c) $90^0$ and $-90^0$

Ans: (a)

(v) Which one of the following statements is not true?
(a) The eigenvalues of the matrix $B = P^{-1}AP$ cannot be the same as those of $A$, when $P$ is any nonsingular matrix.
(b) A matrix of order $n$ will have $n$ eigenvalues, which may not all be distinct (i.e. some eigenvalues may be repeated).
(c) The sum of the eigenvalues of a matrix is equal to the negative of its trace. (The trace of a matrix is equal to the sum of the elements on its main diagonal).

Ans: (a)

(vi) Which one of the following statements is true?
(a) The multiplication of a row vector and a column vector is a column vector
(b) The multiplication of a row vector and a column vector is a square matrix
(c) The multiplication of a row vector and a column vector is a scalar quantity
Ans: (c)

(vii) The roots of the cubic equation
\[ x^3 + 3x^2 + 4x + 2 = 0 \]
are
(a) 2, 3 and 4
(b) -1, -1 +j, -1 -j
(c) 1, 1+j, 1-j
Ans: (b)

(viii) The Laplace inverse transform of
\[ F(s) = \frac{10(s^2 + 2s + 5)}{[(s+1)(s^2 + 6s + 25)]} \]
is
(a) \( e^{-t} + e^{-3t} (8\cos 4t - 6\sin 4t) \)
(b) \( 2e^{-t} + e^{-3t} (3\cos 4t - 4\sin 4t) \)
(c) \( 2e^{-t} + e^{-3t} (8\cos 4t - 6\sin 4t) \)
Ans: (c)

(ix) Which one of the following statements is not true?
(a) The transfer function of a system (or element) does not represent the relationship describing the dynamics of the system
(b) A transfer function may only be defined for a linear, stationary (constant parameter) system
(c) The Laplace transformation may not be utilized for a time-varying system.
(d) The transfer function description does not include any information concerning the internal structure of the system and its behaviour.
Ans: (a)
Which one of the following set of block diagrams is equivalent?

(a) 

(b) 

(c) 

The symbol $\Leftrightarrow$ stands for equivalence.

Ans: (b)

(x) A servo-mechanism is called a proportional error device when the output of the system is

(a) a function of error and the first derivative of error
(b) a function of the first derivative of error only
(c) a function of error only
(d) none of the above.

Ans: (c)

(xi) Fig. 11 shows a differentiating circuit.

\[ \frac{V_2(s)}{V_1(s)} \]

The transfer function of the circuit, \( \frac{V_2(s)}{V_1(s)} \), is

(a) \( \frac{R_2}{1 + s CR_1} \)
(b) \( \frac{s + 1/R_1C}{s + (R_1+R_2)/R_1R_2C} \)
(c) \( \frac{R_1}{1 + s CR_2} \)
(d) \( \frac{s + 1/R_2C}{s + (R_1+R_2)/R_1R_2C} \)

Ans: (b)

(xii) A home heating feedback-control system consists of the valve which controls the gas or power input, the heater, and the thermo-sensor. Which one of the following statements is true?

(a) The heater is in the feedback path
(b) The valve is in the feedback path
(c) The thermo-sensor is in the feedback path
(d) The thermo-sensor is in the forward path

Ans: (c)
(xiii) Match List E that gives the physical element with List F that gives the type of element.

List E- List F

A-Translational spring I-Energy dissipater
B-Rotational mass II-Capacitive storage
C-Thermal resistance III-Inductive storage

The correct matching is

(a) AIII BII CI
(b) AII BI CIII
(c) AI BIII CII

Ans: (a)

(xiv) In general, the programmer does not write the computer program in machine language, because

(a) The program is not executable by the machine.
(b) It is a tedious job for the programmer and is best left to the machine to convert the source program to the machine language program.
(c) There is no need to do it
(d) The machine takes more time to solve the required problem.

Ans: (b)

(xv) Which of the following statements is true?

(a) The FORTRAN compiler changes the object program to the source program.
(b) FORTRAN language is the same as the machine language
(c) It is the object program that is actually executed by the computer to obtain results
(d) The computer executes the source program to obtain the results.

Ans: (c)
(xvi) Programming is a difficult task in one of the following:

(a) Analogue computer
(b) Digital computer
(c) Microprocessor or microcomputer
(d) Minicomputer.

Ans: (c)

(xvi) The number of roots of the polynomial

\[ s^4 + 2s^3 + 4s^2 + 8s + 15 = 0 \]

in the right-half of the s-plane are:

(ii.) Four
(ii) None (two pairs of roots on the jω axis)
(iii.) None (two roots on the jω axis)
(iv.) Two.

Ans: (d)

(xvii) Choose the correct answers from those given below:

i. The following matrix is singular

\[
\begin{bmatrix}
1 & 1 & 2 \\
2 & 0 & 2 \\
3 & 1 & 2
\end{bmatrix}
\]

FALSE

ii. The Laplace transform of

\[ f(t) = \frac{1}{(b-a)} \cdot (b^t - a^t) \]

is

\[ F(s) = \frac{1}{s} \]
The partial fraction expansion of \( F(s) = \frac{s^2 + 2s + 3}{(s+1)^3} \) is

(a) \( \frac{2}{(s+1)^3} + \frac{1}{s+1} \)

(b) \( \frac{2}{(s+1)^3} + \frac{3}{s+1} \)

(c) \( \frac{1}{(s+1)^3} + \frac{2}{(s+1)^2} + \frac{3}{s+1} \)

(d) \( \frac{1}{(s+1)^3} + \frac{3}{s+1} \)

Ans: a

The transfer function, \( \frac{E_o(s)}{E_i(s)} \) of the ideal resonant filter of Fig.12 is

\[ s^2LC \]
\[ s^2LC + 1 \]

(b) Zero
(c) Unity
(d) \( \frac{1}{s^2LC + 1} \)

Ans: (d)

(xx) Modern control theory

(a) is not applicable to multiple-input-multiple-output systems, non-linear systems and time-varying systems
(b) is essentially a time-domain approach, while conventional control theory is a frequency domain approach
(c) does not enable us to design optimal control systems with specific performance indices.
(c) does not enable us to include initial conditions in the design of the control system.

Ans: (b)

(xxii) The resultant equivalent transfer function \( C(s)/R(s) \) of the system shown in Fig.13 is

(a) \( \frac{G_1G_2}{1+G_1G_2H_1} \)
(b) \( \frac{G_1G_2}{1+G_1G_2+G_1G_2H_1} \)
(c) \( \frac{G_1G_2}{1+G_1G_2+G_2H_1} \)
(d) \( \frac{G_1G_2}{1+G_1G_2+G_1H_1} \)

![Diagram of control system](image-url)
Ans: (b)

(xxii) Match List E with List F in the following Table of closed-loop systems and their outputs, \( C(z) \).

<table>
<thead>
<tr>
<th>List E</th>
<th>List F</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td></td>
</tr>
</tbody>
</table>

**List E**

<table>
<thead>
<tr>
<th>( R(s) )</th>
<th>( C(s) )</th>
<th>( C(z) )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**List F**

I

\[ C(z) = \frac{RG(z)}{1+HG(z)} \]

II

\[ C(z) = \frac{G(z)R(z)}{1+G(z)H(z)} \]
C

R(s)  C(s)  C(z)
+

-  G(s)

H(s)

III

C(z) = G(z) R(z)
1 + GH(z)

The correct matching is
(a) AIII BII CI
(b) AI BIII CII
(c) AII BIII CI
(d) AI BII CIII

Ans: (a)

TOP